Variable Step Size LMS Algorithm

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***Abstract*—Through further improvements, a new variable step size LMS adaptive filter algorithm is proposed, which based on the variable step size LMS adaptive filter algorithm of Lorentzian function. This algorithm not only solves the contradiction between the convergence rate and steady-state error, but also improves the anti-interference ability and eliminates the irrelevant noise. Therefore, it provides greater flexibility for the practical application.**

***Index Terms*—Adaptive filter, variable step-size, convergence rate, anti-interference ability, noise cancellation.**

I. Introduction

Widrow and Hoff proposed the least mean square (LMS) algorithm in 1960, in view of its simple structure, low computational complexity and easy implementation, it is widely used in channel equalization, system identification, spectral analysis, signal detection, noise cancellation and beamforming. The iterative formula of fixed step size LMS algorithm is：

$$\mathrm{X}(n) = \begin{bmatrix} x(n) & x(n-1) & x(n-2) & \cdots & x(n-L+1) \end{bmatrix}^T \tag{1}$$

$$e(n) = d(n) - X(n)^T W(n) \tag{2}$$

$$W(n+1) = W(n) + 2\mu e(n) X(n) \tag{3}$$

$X(n)$ is the input signal vector of adaptive filter at n times, $W(n)$ is the estimate value of weights vector of filter, $d(n)$ is the desired signal, $\mathrm{e}(n)$ is the error signal, *L* is the order of devices of the filter; μ is the step factor, which used to control the stability and convergence rate of algorithm. To ensure the convergence of LMS algorithm, the range of step size μ is: $0 < \mu < 1/\lambda\max$, $\lambda\max$ is the largest eigenvalue of the autocorrelation matrix of the input signal. The time constant of adaptive is: $\tau_{mse} = 1/(4\mu\lambda_n)$, the total disorder is: $M = \mu tr(R)$. From the above formulas we can see, when choose the value of the step factor, the steady-state error and convergence rate are contradictory mutually, that is, the smaller step size lead to smaller offset and the slower convergence rate; the larger step size results in faster convergence rate and the larger offset.

II. Improved Variable Step Size LMS Algorithm

In order to overcome these shortcomings, people raised many variable step size LMS algorithms, that is to adjust the step size dynamically during the process of convergence. Document [1] proposed a variable step size LMS algorithm based on sigmoid function, the step size $\mu(\mathrm{n})$ is the sigmoid function of $\mathrm{e}(n)$:

$$\mu(n) = \beta[\frac{1}{1+e^{-\alpha|e(n)|}} - 0.5] \tag{4}$$

It solved the contradiction of the convergence rate, tracking capabilities and the steady-state error, but when the error close to zero, step size μ changes too much, that lead to greater step in steady state. Documents [2] and [3] give the improved algorithms based on document [1], and make the error changes smaller in steady state. Document [4] proposed a variable step size LMS adaptive filter algorithm based on Lorentzian function it uses the Lorentzian function as the formula of step size $\mu(\mathrm{n})$:

$$\mu(n) = \alpha \log[1 + \frac{1}{2}(\frac{e(n)}{\delta})^2] \tag{5}$$

It improved the convergence speed and tracking speed further. In this paper, a new adaptive variable step size algorithm is proposed based on the LMS filter algorithm of Lorentzian function, it eliminates the influence of irrelevant noise, and has good anti-interference ability, the formula of improved step factor $\mu(\mathrm{n})$ is:

$$\mu(n) = \alpha \log[1 + \frac{1}{2}\frac{e(n)e(n-1)}{\delta^2}] \tag{6}$$

III. Theoretical Analysis of Improved Algorithm

A. Convergence analysis

To make this algorithm converge, step factor $\mu(\mathrm{n})$ should be satisfied that $0 < \mu(n) < 1/\lambda\max$. Variable step size factor $\mu(\mathrm{n})$ is a positive value which is under the control of parameters α , δ and the correlational values of the instantaneous error. In the beginning of adaption, the big error makes the big step size and fast convergence rate; with the adaptive process going on, errors are reduced gradually, step size is also decreased. When adaption reached steady-state phase, the error is very small, and close to zero, step size is zero approximately, so generate a small offset in the vicinity of the best weight. Parameter α controls the convergence rate, the bigger the value is, the faster the convergence rate. Parameter δ is used to control the changes of step size in the steady-state when the error is close to zero, so the step size changes slowly in the vicinity of the best step, and avoids large changes generated by the

Manuscript received April 17, 2012; revised June 3, 2012.

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independent noise. The method of how to select the values of α and δ refer to the document [4], this article is no longer described in detail.

Anti-interference analysis

The formula of error $e(n)$ is: $e(n)=d(n)-XT(n)W(n)$, which can be rewritten as:

$$\mathrm{d}(n)=e(n)+X^{T}(n)W(n) \tag{7}$$

As the error signal $e(n)$ and input signal $X(n)$ are related, we can transform the expected signal into another form in order to analysis conveniently:

$$d(n)=X^{T}(n)W^{*}(n)+N(n) \tag{8}$$

where $N(n)$ is the interference noise, generally Gauss white noise, and independences with the input signal; $W^{*}(n)$ is the best weights vector. We assume that:

$$\Delta W(n)=W(n)-W^{*}(n) \tag{9}$$

$\Delta W(n)$ is the deviation of weights vector, so we can receive that:

$$e(n)=N(n)=X^{T}(n)\Delta W(n) \tag{10}$$

$$e^{2}(n)=N^{2}(n)-N(n)X^{T}(n)\Delta W(n)$$

$$-X^{T}(n)\Delta W(n)N(n)+X^{T}(n)\Delta W(n)X^{T}(n)\Delta W(n) \tag{11}$$

$$e(n)e(n-1)=$$

$$N(n)N(n-1)-N(n)X^{T}(n-1) \tag{12}$$

$$\Delta W(n-1)-X^{T}(n)\Delta W(n)N(n-1)+X^{T}(n)\Delta W(n)X^{T}(n-1)\Delta W(n-1)$$

As the noise signal $N(n)$ is independent with the input signal and its mean is zero, take expectations both on the formula (10) and formula (11), we can receive the reduction formula:

$$E[e^{2}(n)]=E[N^{2}(n)]+E[X^{T}(n)\Delta W(n)X^{T}(n)\Delta W(n)] \tag{13}$$

$$E[e(n)e(n-1)]=E[X^{T}(n)\Delta W(n)X^{T}(n-1)\Delta W(n-1)] \tag{14}$$

According to the view of statistics, it can be seen from the formula (12) and (13), in formula (13), there is $E[N^{2}(n)]$, that is to say the expectation of $e^{2}(n)$ is related to the interference signal $N(n)$. Therefore, in the case of larger interference, the stability of algorithm will be influenced more or less. In formula (14), the expectation of $e(n)e(n-1)$ is only related to the input signal $X(n)$, and independent with the noise signal $N(n)$, that is the adaptive algorithm has stronger anti-interference ability. Compared with the previous algorithm, the step iterative algorithm of this paper has smaller steady-state error, and also has a good performance especially in low *SNR* conditions.

IV. Computer Simulation

To test the convergence rate, steady-state error, anti-interference ability and other properties, we use adaptive noise canceling system for the computer simulation. The adaptive noise cancellation system is shown in figure 1. The original sinusoidal input signal $X(n)=\sin(2\pi n/10)$ in the main input port, the interference signal is $\mathrm{n}_0(n)$ which is white noise, and SNR is 20dB, the polluted signal $d(n)$ is composed of $X(n)$ and $\mathrm{n}_0(n)$. In the reference channel, we input other noise signal $\mathrm{n}_1(n)$ which is related to $\mathrm{n}_0(n)$, then outputs $\mathrm{y}(n)$ through the adaptive filter, by adjust the value of $\mathrm{e}(n)$ which is the deference between $d(n)$ and $\mathrm{y}(n)$ to adjust the coefficients of adaptive filter automatically, in order to filter the noise in original signal.

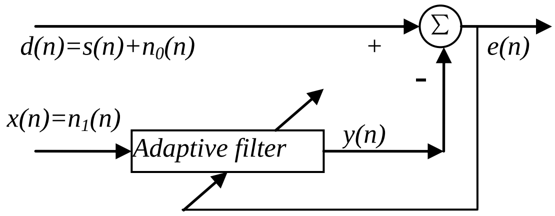

Fig. 1. Adaptive noise cancellation system.

We use the improved adaptive LMS algorithm proposed in this paper for the adaptive filter in Fig. 1. In order to get each curves, the sampled total points is setted as 100, filter's order is 2, simulate 20 times, and then obtain the statistical average to seek their learning curves. The parameters setting in document [4] is: α = 0.05, δ = 0.01; in this paper is set to: α = 0.05, δ = 0.01.

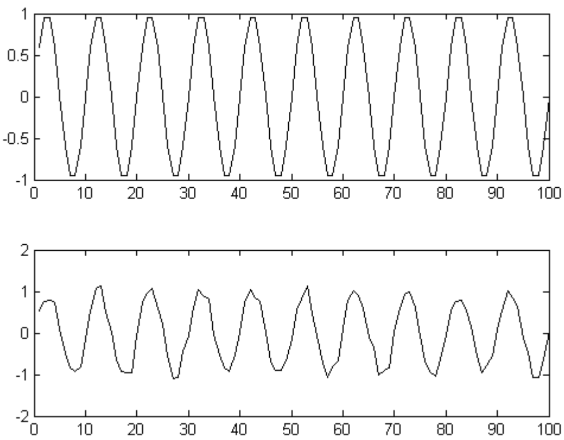

Fig. 2. Waveform of original signal and noise signal.

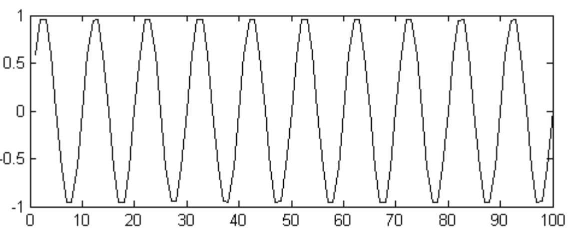

Fig. 3. The filtered waveform of the proposed algorithm.

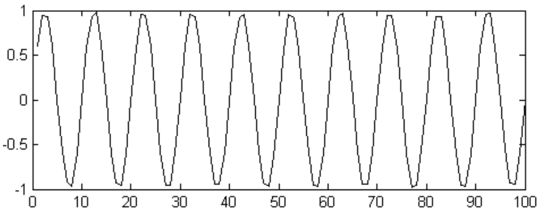

Fig. 4. The filtered waveform in document [4].

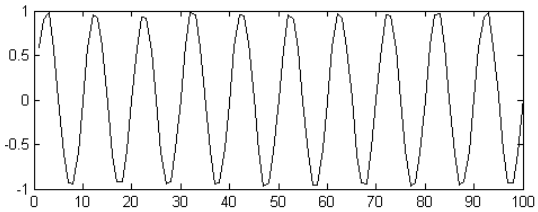

Fig. 5. The filtered waveform of the fixed step size LMS algorithm.

Contrast the waveform of Fig. 3, Fig. 4 and Fig. 5, in Fig. 3, the waveform is the closest to the original signal waveform, and the distortion is minimum; in Fig. 4, the waveform distortion at the peaks are obvious, and the distortion is low relatively; in Fig. 5, the waveform distortion is the most serious. It can be drawn that this method of filtering is better than the algorithm pre- improved filter.

To verify the algorithm's anti-interference capability, we make the learning curves of this paper’s algorithm and document [4] under different SNR. The parameters setting are the same as above, the SNR from top to bottom are: 10 dB, 20 dB and 30 dB which are shown in Fig. 6 and Fig. 7.

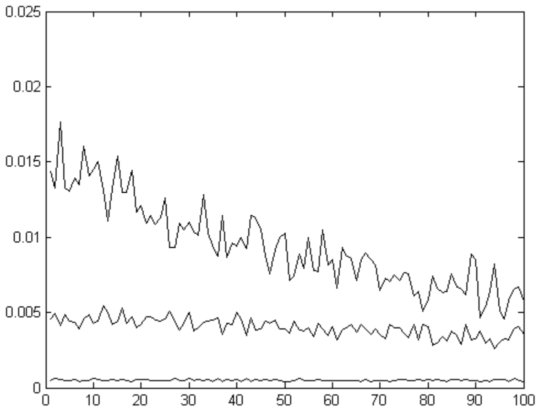

Fig. 6. The learning curves for different *SNR* of the proposed algorithm.

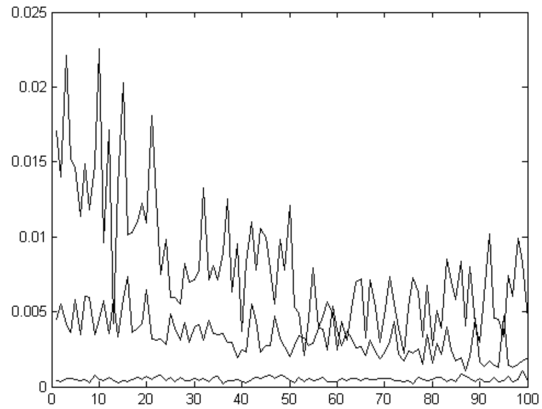

Fig. 7. The learning curves for different SNR in document [4].

Compare Fig. 6 and Fig. 7, the curve in Fig. 6 is smoother than in Fig. 7, and has better convergence rate, smaller steady-state offset at the signal to noise and lower ratio. Therefore, in low SNR conditions, the anti-interference and outperform in this paper is better than in the document [4].

V. Conclusion

We improved the algorithm base on document [4] in this paper. It ensure the fast convergence rate and low steady-state error, while inhibit the noise not related to input signal effectively. Especially in low *SNR* conditions, the effect of filter is better for signal filtering and extraction for weak signal detection under the conditions of relatively low signal to noise ratio of signal filtering and extraction.

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