Crystal Fiber Based Erbium Doped Amplifiers and Their Gain

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Abstract—The paper is based on the numerical solution of Euler’s method to deal with the initial value problems for Erbium doped fiber amplifiers’ (EDFAs) rate equations, and their gain dependences are discussed. Simulation of the propagation of the signal and pump power versus the amplifier length has been done. Modifying the input pump power and input signal power has obtained the gain profile of the PCF based EDFAs. Fiber designs which allow us to have large gain are presented and demonstrated that the PCF based Erbium doped amplifiers may deliver gain 54dB with small losses. The gain is greater than the conventional EDFAs.

Index Terms—Photonic crystal fiber, erbium doped fiber amplifiers, gain.

I. INTRODUCTION

The term “amplifiers” refer to boost its transmission of information along the fiber. Over a certain distance, information cannot be transmitted through the fiber without optical amplifiers. Fiber attenuation limits the strength of optical signal. Power amplifiers are employed to enlarge the power of the signal before it approaches the fiber transmission link and it can extend the transmission distance. Power amplifier has high gain while preamplifier needs lower noise value and the line amplifier requires both of them. Erbium—doped fiber amplifiers (EDFAs) can be made as optical amplifiers. After the first paper on EDFAs published in 1987, optical communications has begun its new era. EDFAs become common and popular now, because Erbium atoms provide light amplification over the range of wavelength 1525—1610 nm.

The objective of this paper is to develop a practical method to perform the gain for Erbium Doped Fiber Amplifiers based on Photonic Crystal Fibers on the pump and signal power in co-propagating pump configuration. Simulation is based on the numerical solution of Euler’s method to deal with the initial value problems for Erbium doped fiber amplifiers rate equations and gain dependences will be discussed. The paper interprets the theory of the erbium doped fiber amplifiers and analyzes photonic crystal fiber based erbium doped amplifiers. Simulation the propagation of the signal and pump power versus the amplifier length is to be done in this project.

II. ERBIUM-DOPED FIBER AMPLIFIERS (EDFAs)

A. Three Levels Erbium-Doped Fiber Amplifiers (EDFAs)

The decay time from level 1 to the ground state normally is very short and can be neglected. So the rate equations for the three level N3, N2 and N1, are presented as below:

\[
dN_3 \over dt = -N_3 \over \tau_{21} + (N_1 - N_3)\phi_p\sigma_p
\]

(1)

\[
dN_2 \over dt = -N_2 \over \tau_{21} + N_3 \over \tau_{32} - (N_2 - N_1)\phi_s\sigma_s
\]

(2)

\[
dN_1 \over dt = N_2 \over \tau_{21} - (N_1 - N_3)\phi_p\sigma_p + (N_2 - N_1)\phi_s\sigma_s
\]

(3)

Fig. 1. The Three Level Energy Structure of the Erbium Er3+ Doped Fiber Amplifier.

Erbium is a rare earth element that is used for amplification operating around 1550 nm which is the wavelength for long haul communication system. Erbium doped optical silica-glass SiO2 fiber makes EDFAs which are widely employed in optical communications. EDFAs which are using a unique, proprietary rare earth doping method and meet the requirements of high data transmission rates over long distances. They have played a crucial role in the advancement of communication system. The doped fiber is a gain medium in which signal and pump wavelengths are injected, and high-quantum-efficiency, high-rare earth concentration are produced. Amplification is obtained by interaction of wavelength and doping ions. The key physical phenomenon behind signal amplification is stimulated emission of radiation by atoms in the presence of an electromagnetic field. An optical signal in the case of optical amplifier is the field. For amplification to occur, population inversion has to be achieved by supplying additional energy in a suitable form to pump the electrons to the higher energy level. But to understand the impact of spontaneous emission is important.
B. Cross Sections

The emission cross section and the absorption cross section are related to the frequency or the wavelength. They express the abilities of an ion to either emit or absorb photon. If there are two energy levels E₁ and E₂, the energy difference is then the higher energy minus the lower energy E₂-E₁. And it is proportional to absorption cross section σ₂₁ when the transition happens from level 1 to level 2. The emission cross section σ₂₁ corresponds to the transition which takes place from level 2 to level 1 and is also proportional to the energy difference E₂-E₁. The relationship between the emission cross section and the absorption cross section is described by

\[ \sigma_{21} = \sigma_{12} (\nu) e^{(g-h\nu)/kT} \]  

where g is the average energy of the manifolds.

C. Amplifier Gain

1) Stimulated emission is the process that causes the optical amplification. The optical gain G would take the form

\[ P_{\text{out}}(t) = GP_{\text{in}}(t) \]  

There are three classical ways of providing optical gain:

1) Optical power amplifier (or booster amplifier): amplifier immediately after transmitter to effectively increase transmitted power.

2) Optical preamplifier: amplifier immediately before receiver to increase received power and thus aid detection. It also effectively improves receiver sensitivity.

3) Optical in-line amplifier: amplifier periodically placed along the transmission line to compensate for fiber loss and often arranged in either pre- or booster configurations.

But it is more interesting to study the optical amplifier gain in the optical fibers. Assume that the fiber length is L, N is the density of photons. γ(ν) is the gain. Figure 2 states the amplifier gain at z position in the fiber.

\[ d\phi = N*W*dz*W = \phi*\sigma(\nu) \Rightarrow d\phi = N*\phi*\sigma(\nu)*dz \]  

\[ \frac{d\phi(Z)}{dz} = \gamma(\nu)\phi(Z) = N\sigma(\nu)\phi(z) \]  

The gain of the signal and the pump are yielded from:

\[ g_s = (N_2 - N_1)\sigma_s \]  

\[ g_p = (N_3 - N_1)\sigma_p \]  

Considering a situation where the erbium ions are uniformly doped in the core of the PCF, the propagation equations which demonstrate the behavior of the erbium-doped fiber amplifiers can be rewritten as:

\[ \frac{dI_s(Z)}{dz} = \frac{1}{1 + \Gamma_s(Z)/\Gamma_{\text{sat}}(Z)} (I_s(Z) - 1) I_s(Z)\sigma_s N \]  

\[ \frac{dI_p(Z)}{dz} = -\frac{1}{1 + 2\eta I_s(Z) + \Gamma_p(Z)} I_p(Z)\sigma_p N \]  

where \( I_s = \frac{I_s}{I_{th}} \); \( I_p = \frac{I_p}{I_{th}} \); are the normalized intensities in the units of the pump threshold. Where \( \sigma_s \) denotes absorption cross section when the transition happens from level 2 to level 1, and \( \sigma_p \) corresponds to the transition which takes place from level 1 to level 3.

Consider the field propagating along the fiber, take account infinitesimal length dz, and combine effects of pumping and stimulated emission:

\[ \frac{d\phi}{dz} = \phi_s g_s \]  

\[ \frac{d\phi_p}{dz} = \phi_p g_p \]  

where \( \phi_s = \frac{I_s}{h\nu_s} \); \( \phi_p = \frac{I_p}{h\nu_p} \); \( \nu_p \) and \( \nu_s \) are the frequencies of the pump and signal respectively. Eq. (8) and (9) will be modified as:

\[ \frac{dI_s}{dz} = (N_2 - N_1)\sigma_s I_s \]  

\[ \frac{dI_p}{dz} = (N_1 - N_1)\sigma_p I_p \]  

D. Crystal Fiber Based EDFAs

For the photonic crystal based erbium doped holey core fiber amplifier, the contribution of the erbium ions is the complex refractive index in the core area. When a field propagates along the z axis in an amplifier, it can be described as:

\[ E = E_0 \exp(j(\omega t - \beta z)) = E_0 \exp(j\omega t) \exp(-\beta z) \]  

The wave equation is expressed as:

\[ \nabla^2 E + n^2k_0^2E - \beta^2 E = 0 \]  

Considering a situation where the erbium ions are uniformly doped in the core of the PCF, (10) and (11) will be changed as:

\[ \frac{dI_s(Z)}{dz} = \frac{\Gamma_s(Z)}{1 + \Gamma_s(Z)/\Gamma_{\text{sat}}(Z)} (I_s(Z) - 1) I_s(Z)\sigma_s N \]  

\[ \frac{dI_p(Z)}{dz} = -\frac{\Gamma_p(Z)}{1 + 2\eta I_s(Z) + \Gamma_p(Z)} I_p(Z)\sigma_p N \]  

where \( I_p = \frac{I_p}{I_{th}} \); \( I_s = \frac{I_s}{I_{th}} \); \( \eta = \frac{h\nu_p}{h\nu_s} \); \( \Gamma_{\text{sat}}(Z) = \frac{1 + I'_s(Z)}{2\eta} \); \( \Gamma_s \) and \( \Gamma_p \) are the confinement factors of the signal and pump respectively.
E. Euler Method for Photonic Crystal EDFAs

![Diagram](Euler Method for Photonic Crystal EDFAs)

To solve the equations (14) and (15), one can use the Euler’s method. Assume that equations have their unique solution and two continuous derivatives along the fiber length $L$, so that for each $k=0, 1, 2, 3, ..., M-1$,

$$I_s(z_{k+1}) = I_s(z_k) + \Delta z \frac{d^2 I_s(z_k)}{dz^2} + \Delta^2 z \frac{d^2 I_s(z_k)}{dz^2}$$  \hspace{1cm} (18)

$$I_p(z_{k+1}) = I_p(z_k) + \Delta z \frac{d^2 I_p(z_k)}{dz^2} + \Delta^2 z \frac{d^2 I_p(z_k)}{dz^2}$$  \hspace{1cm} (19)

$$z = z_0 + k \Delta z$$, for each $k=0, 1, 2, 3...M$.

Hence, within the Normalized equations for PCF based erbium doped fiber amplifiers, choose proper parameters and initial values of signal and pump power, then exploit Euler’s method, the output accurate value can be found.

III. MODELING RESULTS

To demonstrate its effectiveness in the modal and the gain of the PC based EDFAs, the intersection number is 1000 and the mesh size is $1 \mu$m with the calculation window of 72 x 72 meshes. Both the confinement factors of the pump and signal are greater than 0.8. From the simulations, it can be found that if the initial pump power is less than 10dBm (see Fig4), the gain along the amplifier length will not be stable and there is a saturation gain existed. For the Er$^{3+}$ concentration $N=0.7 \times 10^{19}/cm^{-3}$ we find the optimized gain is $G=54.813 \text{dB}$, for initial pump power $Pp=50\text{mW}(17\text{dBm})$ and the initial signal power is $Ps=-40\text{dBm}$; see figure 5. The optimal fiber length $L=14.7\text{m}$. When $L=15\text{m}$, the gain is $54.808\text{dB}$. Pump power can boost the signal quickly and keep this amplification almost unchanged. In Ref. [5] for $N=0.2 \times 10^{19}/cm^{-3}$, it reported that the optimal fiber length is 18.6m; the optimized gain is $G=41.9 \text{dB}$ under the same initial signal and pump power conditions.

![Graphs](Graphs)

IV. CONCLUSION

In this paper, it is found that the hollow-core PCF based erbium doped amplifiers of the length 15m, the relative core size $d/\Lambda=0.33$, when the input signal is -40dBm, the input...
pump power 50mW is 17dBm, the gain of this amplifier is obtained 54.81dB. The calculation window is used as 72μm×72μm, the PCF core size 6μm. It will be very interesting by using full vector method to calculate the solid core PCF based EDFAs gain profile. It is important to point out that the very promising properties of the PCF gives some new ideas, i.e. the PCF based Erbium Doped amplifiers would be the novel devices in the telecommunication systems. By employing Euler’s methods, the gain profiles of the PCF based erbium doped amplifiers have been reported.

ACKNOWLEDGEMENTS

I would like to thank my supervisor Dr. S. Sujecki in Nottingham University for providing me help, support and encouragement during this paper. His advice and suggestions has been invaluable.

REFERENCES