Topological Properties of few Poly Oxide, Poly Silicate, DOX and DSL Networks

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***Abstract*—This paper deals with a new interconnection network motivated by molecular structure of a chemical compound SiO_4. The different forms of silicate available in nature lead to the introduction of the (DSL) dominating silicate network. The first section deals with the introduction to (Resolving number) minimum metric dimension problems, and few related work about Silicate networks. The second section introduces and gives an account of the proof to the topological properties of poly-oxide, poly-silicate, dominating oxide (DOX), dominating silicate networks, and regular triangulene oxide network (RTOX). The third section deals with the drawing algorithm for dominating silicate network, and shown complete embedding of oxide, silicate network in to dominating oxide, dominating Silicate network respectively. The fourth section contains the proof of the minimum metric dimension of regular triangulene oxide network to be 2.**

***Index Terms*—Dominating oxide, dominating silicate, embedding, minimum metric dimension, mesh like architectures, poly-oxide, poly-silicate networks, topological properties.**

I. Introduction

A fixed interconnection parallel architecture is characterized by a graph, with vertices corresponding to processing nodes and edges representing communication links [15]. Interconnection networks are notoriously hard to compare in abstract terms [5], [9], [13]. Researchers in parallel processing are thus motivated to propose new or improved interconnection networks, arguing the benefits and offering performance evaluations in different contexts [2], [4], [5] , [7], [9], [12], [14]. A few networks such as Hexagonal, Honeycomb, and grid networks, for instance, bear resemblance to atomic or molecular lattice structures. *Honeycomb networks*, built recursively using the hexagon tessellation [12]-[15], are widely used in computer graphics [10], [15] cellular phone base station [11]-[15], image processing[3], [15], and in chemistry as the representation of benzenoid hydrocarbons [13] and Carbon Hexagons of Carbon Nanotubes [8]. *Hexagonal networks* are based on triangular plane tessellation, or the partition of a plane into equilateral triangles [4], [11], [14], [15]. Hexagonal network represents a host cyclotriveratrylene with halogenated mono carbaborane anions [1] and Silicon Carbide [15]. C*arbon nanotubes* consist of shells of *sp* -hybridized carbon atoms forming a hexagonal network, arranged,

Helically within a tubular motif [1] in this paper, we introduce *dominating silicate networks*. Silicates are obtained by fusing metal oxides or metal carbonates with sand. Essentially all the silicates contain SiO_4 tetrahedra.

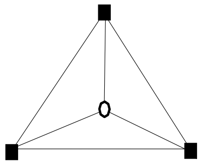

Fig. 1. SiO_4 tetrahedra where the corner vertices represent oxygen ions and the center vertex the silicon ion.

The corner vertices of SiO_4 tetrahedron represent oxygen ions and the center vertex represents the silicon ion. Graph theoretically, we call the corner vertices as *oxygen nodes* and the center vertex as *silicon node*. See Fig. 1. The minerals are obtained by successively fusing oxygen nodes of two tetrahedra of different silicates. Here ,We study the topological properties of Poly Oxide ,Poly Silicate , *DOX*, *DSL* networks as it has been studied for other networks[2]-[15]. We study its structure and properties from the perspective of computer Science.

A. An Overview of this Paper

The first paper on the notion of a resolving set appeared as early as 1975 under the name '*locating set*'[28] . Slater [28], [29] introduced this idea to determine uniquely the location of an intruder in a network[31]. Harary and Melter [21] and Khuller et al. [23] discovered this concept independently and used the term *metric basis*. They called the resolving number as *minimum metric dimension*. This concept was rediscovered by Chartrand et al. [30] and also by Johnson [22] of the Pharmacia Company while attempting to develop a capability of large datasets of chemical graphs. It was noted in [20] that determining the minimum metric dimension problem (resolving number) of a graph is an *NP*-complete problem. It has been proved that this problem is *NP*-hard [23] for general graphs. Manuel et al. [24] have shown that the problem remains *NP*-complete for bipartite graphs. This problem has been studied for trees, multi-dimensional grids [23], Petersen graphs [3], torus networks [27], Benes networks [24], honeycomb networks [25], enhanced hyper cubes [18], Illiac networks [19] and X-trees[15]. In this paper We have proved that Minimum metric dimension of *Regular Trianguline Oxide network RTOX(n)* is 2. There are many applications of minimum metric dimension to problems of network discovery and verification [32], pattern recognition , image processing and robot navigation [16], geometrical routing protocols [33], connected joins in graphs[34], coin weighing problems[35]

B. Properties of Silicate and Oxide Network

A silicate network can be constructed in different ways.

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Consider a honeycomb network $HC(n)$ of dimension n. Place silicon ions on all the vertices of $HC(n)$. Subdivide each edge of $HC(n)$ once. Place oxygen ions on the new vertices. Introduce $6n$ new pendant edges one each at the 2-degree silicon ions of $HC(n)$ and place oxygen ions at the pendent vertices. See Figure 2(a). With every silicon ion associate the three adjacent oxygen ions and form a tetrahedron as in Fig.2(b). The resulting network is a silicate network $SL(n)$. The parameter n of $SL(n)$ is called the *dimension* of $SL(n)$. The graph in Figure 2(b) is a silicate network of dimension 2[15].

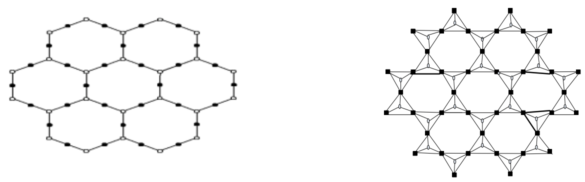

Fig. 2(a) Fig. 2(b)

Fig. 2. Construction of silicate network $SL(n)$ from $HC(n)$

Theorem 1.1: The number of nodes in $SL(n)$ is $15n^2+3n$, and the number of edges of $SL(n)$ is $36n^2$.

When all the silicon nodes are deleted from a silicate network, we obtain a new network which we shall call as an *Oxide Network*. See Fig. 3. An n-dimensional oxide network is denoted by $OX(n)$.

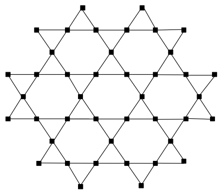

Fig. 3. An *oxide network* $OX(2)$

II. Topological Properties

Theorem2.1: Let G be a simple undirected graph with $\gamma(G)$ of vertices and $e(G)$ edges. Let $L(G)$ be the line graph of G then

1) Number of vertices of $L(G) = \gamma[L(G)] = e(G)$
2) Number of edges of L(G) is

$$e[L(G)] = \{ \sum [d_G(x)]^2 /2\} - e(G). \quad [9]$$

Theorem2.2: The number of nodes in $OX(n)$ is $9n^2+3n$ and edges $18n^2$.

Proof: Let $G = H(n)$. From the drawing algorithm of silicate /Oxide network, $\gamma\ [OX(n)]$ = Number of edges of $HC(n)$ + Number of pendent edges introduced for $HC(n)$.

$$= 9n^2 - 3n + 6n$$
$$= 9n^2 + 3n$$

For HC(n), $\sum[d_G(x)]^2$ = (number of vertices of degree 2) $\times(2^2)$ + (number of vertices of degree 3) $\times(3^2)$

$$= 6n\times(2^2) + (6n^2-6n)\times(3^2)$$
$$= 24n+54n^2-54n$$
$$=54n^2-30n$$

$$e\{L(HC(n))\} = \{\sum[d_G(x)]^2/2\} - e(G)$$
$$= \{(54n^2-30n)/2\}-(9n^2-3n)$$
$$= 18n^2-12n$$

$e(OX(n))$ = $e\{L[HC(n)]\}$ +2(Number of pendent edge introduced)

$$= 18n^2-12n+ 2(6n)$$
$$= 18n^2$$

Theorem2.3: The number of nodes in *Single Oxide chain* $OX(1, n)$ is $2n+1$ and edges is $3n$,where n is the number of edges in a row line.

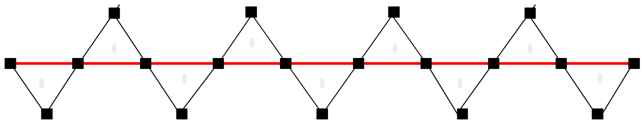

Fig. 4. Single oxide chain $OX(1,9)$ Row line is highlighted with red color.

Theorem2.4: The number of nodes in *Single Silicate chain* $SL(1, n)$ is $3n+1$ and edges is $6n$,where n is the number of edges in a row line.

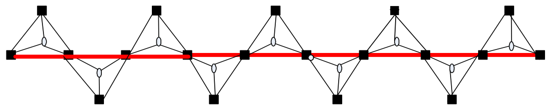

Fig. 5. Single silicate chain $SL(1,9)$ row line is highlighted with red color.

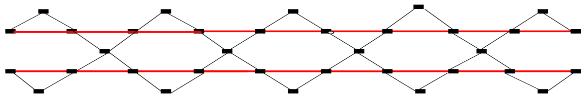

Fig. 6. Oxide double oxide chain $OX(2,9)$

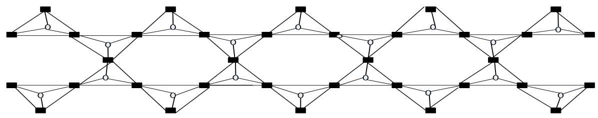

Fig. 7. Double silicate chain $SL(2,9)$

Theorem2.3: The number of nodes in $ROX(n)$ $ROX(m,n)$ is $\gamma = m(2n+1) - \{\lceil \frac{(n-1)}{2} \times \lfloor \frac{m}{2} \rfloor + \frac{(n+1)}{2} \times \lfloor \frac{(m-1)}{2} \rfloor \rceil\}$, $m>2$ and edges $e=3mn$, where m is number of row lines and n is number of edges in a row line.

Proof: It is easy to see that the number of edges $=3\times$number of row line $\times$number of edges in a row line = $3mn$.

Number of vertices = Number of edges – Number of vertices in each row line except corner vertices – number of vertices linked with other oxide chain.

$$=3mn - m(n-1) - \{\lceil \frac{(n-1)}{2} \times \lfloor \frac{m}{2} \rfloor + \frac{(n+1)}{2} \times \lfloor \frac{(m-1)}{2} \rfloor \rceil\}$$

$$=2mn + m - \{\lceil \frac{(n-1)}{2} \times \lfloor \frac{m}{2} \rfloor + \frac{(n+1)}{2} \times \lfloor \frac{(m-1)}{2} \rfloor \rceil\}$$

$$= m(2n+1) - \{\lceil \frac{(n-1)}{2} \times \lfloor \frac{m}{2} \rfloor + \frac{(n+1)}{2} \times \lfloor \frac{(m-1)}{2} \rfloor \rceil\}, m>2$$

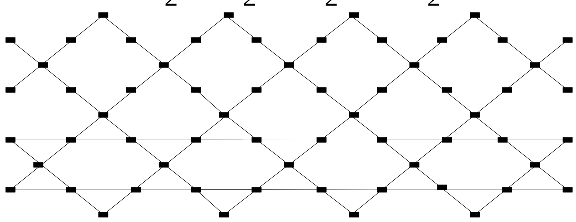

Fig. 8. $ROX(4,9)$

Theorem 2.4: For Rectangular silicate $RSL(m, n)$ $\gamma = m(3n+1) - \{\lceil \frac{(n-1)}{2} \times \lfloor \frac{m}{2} \rfloor + \frac{(n+1)}{2} \times \lfloor \frac{(m-1)}{2} \rfloor \rceil\}$, $m>2$

$e=6mn$ Where m is number of row lines and n is number of edges in a row line.

Proof: Number of nodes = number of nodes in *Rectangular oxide* + Centroid vertex of each K_3 sub graph.

$$= m(2n+1) - \{ \lceil \frac{(n-1)}{2} \times \lfloor \frac{m}{2} \rfloor + \frac{(n+1)}{2} \times \lfloor \frac{(m-1)}{2} \rfloor \rceil \} + mn$$

$$= m(3n+1) - \{ \lceil \frac{(n-1)}{2} \times \lfloor \frac{m}{2} \rfloor + \frac{(n+1)}{2} \times \lfloor \frac{(m-1)}{2} \rfloor \rceil \}, m > 2$$

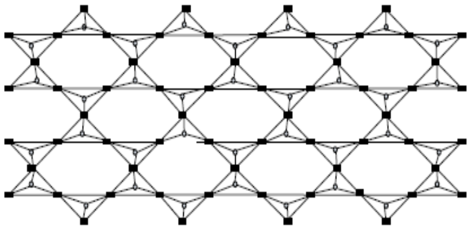

Fig. 9. *RSL*(4,9)

III. Drawing Algorithm for Dominating Silicate Network (*DSL*) From *HC* (*n*)

Step 1: Consider a honeycomb network *HC*(*n*) of dimension *n*.

Step 2: Subdivide each edge of *HC* (*n*) once. Place oxygen ions on the new vertices.

Step 3: In each hexagon cell, connect the new nodes by an edge if they are at a distance of 4 units within the cell.

Step 4: Place Oxygen irons to new edge crossings.

Step 5: Removing the nodes and edges of HC (*n*)**,** we get Dominating Oxide Network.

Step 6: Place a Silicon node to Centroid of each regular sub graph K_3 of Dominating Oxide network and connect it with other oxide node in the same K_3. Thus we get Dominating Silicate network.

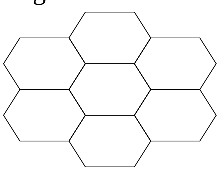

Fig. 10(a). Step-1

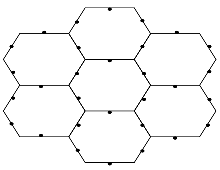

Fig. 10(b). Step 2

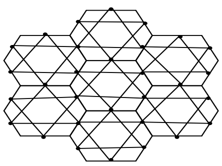

Fig. 11.(a) Step-3

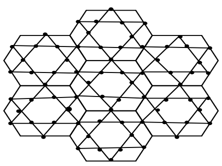

Fig. 11.(b) Step-4

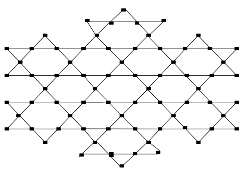

Fig. 12(a). DOX(2)
Step-5

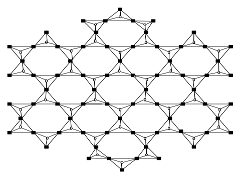

Fig. 11(b). *DSL(2)*
Step-6

Theorem3.1: The number of nodes in *Dominating Oxide network DOX* (*n*) is $27n^2$-21*n*+6 and edges is $54n^2$-54*n*+18.

Proof: From the constructing of *Dominating Silicate network,*

γ [*DOX*(*n*)] = Number of edges in HC(*n*)+Number of new vertices

= *e*[*HC*(*n*)]+6 (*f*-1)(Each bounded dual of *HC*(*n*) we add 6 nodes)

= $9n^2$-3*n* +6($3n^2$-3*n*+2-1)

= $9n^2$-3*n*+$18n^2$-18*n*+6

= $27n^2$-21*n*+6

e[*DOX*(*n*)]=Number of bounded face *HC*(*n*) × Number of edges in *DOX*(1)

= ($3n^2$-3*n*+1) ×(18)

= $54n^2$-54*n*+18.

Theorem 3.2: The number of edges in *Dominating silicate network DSL*(*n*) *is* $108n^2$-108*n*+36 and nodes is $45n^2$ -39*n*+12.

Proof: Number of nodes = Number of nodes *DOX*(*n*) + 1 ×Number of triangle face in *DOX*(*n*)

= ($27n^2$-21*n*+6)+ 6(f-1 in *HC*(*n*))

= ($27n^2$-21*n*+6)+6($3n^2$-3*n*+2-1)

= ($27n^2$-21*n*+6)+$18n^2$-18*n*+12-6

= $45n^2$ -39*n*+12.

Number of edges =3 ×6 ×(f-1in *HC*(*n*)) + edges in *DOX*(*n*)

= 18 ×($3n^2$-3*n*+1) +$54n^2$-54*n*+18

= 54 n^2-54*n*+18 +$54n^2$-54*n*+18

= $108n^2$ -108*n*+36

A. Embedding of Oxide and Silicate in to DOX and DSL

Theorem 3.3: Any *Oxide, Silicate network* of dimension *n* can be embedded in to *DOX*(*n*), *DSL*(*n*) respectively with *dilation* one .

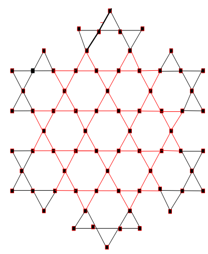

Fig. 13 (a)
Embedding of *OX*(2) in *DOX*(2)

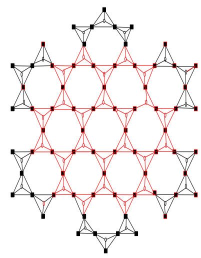

Fig. 13 (b)
Embedding of *SL*(2) in *DSL*(2)

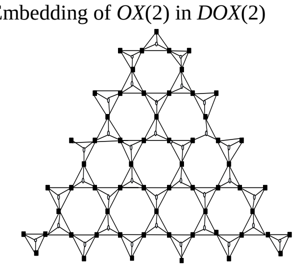

Fig.14(a). *RTSL*(5)

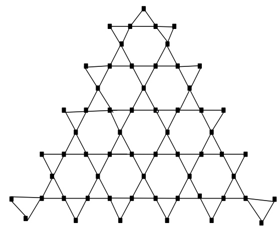

Fig. 14(b). *RTOX*(5)

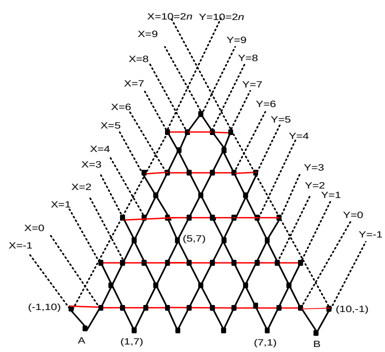

Fig. 15. Coordinate system for *RTOX*(5)

Theorem 3.4: The number of nodes in Regular Triangulene Oxide network *RTOX*(*n*) is($3n^2$ +9*n*+2)/2 , edges is $3n^2$ +6*n* , and triangle face is n^2 +2*n* where n is number of row lines .

Proof : Number of vertices= number of vertices lies in row line and not lies in row line .

= [1+2+3+…+(n+1)]+[4+6+8+…+2(n+1)]
= [1+2+3+…+(n+1)]+2[1+2+3+…+(n+1)]-2
= 3[1+2+3+…+(n+1)]-2
= 3[(n+1)(n+2)/2]-2
= $(3n^2+9n+2)/2$.

Number of edges = 3 × Number of edges in a row lines
= 3 ×[3+ 5+7+…+ (2n+1)]
= 3 ×[1+3+ 5+7+…+ (2n+1)]-3
= $3\times(n+1)^2$-3
= $3\times[n^2+2n+1]$-3
= $3n^2+6n+3-3$
= $3n^2+6n$

By Euler's formula for planar graph,
Face (f) = Number of Edges (e) - Number of Vertices(γ) + 2
= $(3n^2+6n)-(3n^2+9n+2)/2+2$
= $(3n^2+3n+2)/2$

Triangle face = (f-1)-Hexagon face
= (f-1)-[1+2+3+…+(n-1)]
= {[($3n^2+3n+2$)/2]-1}-[n(n-1)/2]
= [$3n^2+3n+2-2-n(n-1)$]/2
= ($3n^2+3n-n^2+n$)/2
= n^2+2n.

Theorem 3.5: The number of nodes in Rhombus Oxide network *RHOX* (n) is $3n^2+2n$ and edges is $6n^2$ where n is the number of corner vertices in a side.

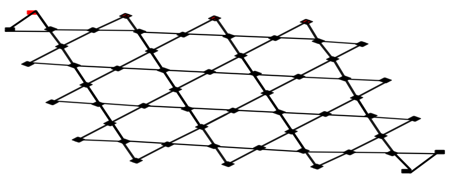

Fig. 16. *RHOX*(4)

Proof: Number of vertices = 2*(Number of vertices above diagonal line) + number of vertices lies in diagonal line

=2 ×{2n+ (2n-2) +(2n-4)+…+[2n- (2n-2)]} +2{1+2+3+…+(n-1)} +n
=2 ×{2n +2n(n-1)-2[1+2+3+…+(n-1)] +2 ×[(n-1)n/2] +n
= 2 ×{$2n+2n^2-2n$ -2 ×(n-1)n/2 } +n^2- n +n
=$4n^2-2(n^2-n)+n^2$
=$3n^2+2n$.

Number of edges
= 2 ×3 ×{(2n-1)+ (2n-3)+(2n-5)+…+2[2n-(2n-1)]}
= 6 ×{2n ×n –[(1+3+5+…+(2n-1)]}
= 6 ×{$2n^2-n^2$}
= $6n^2$

IV. Minimum Metric Dimension of Regular Triangulate Oxide Network

Now let us prove that the *Minimum metric dimension* of *Triangulene oxide network* is 2. It is interesting to learn that [16] a graph has metric dimension 1 if and only if it is a path. Therefore metric dimension of *Triangulene oxide network* is greater than one. In order to exhibit a metric basis of cardinality 2, we require the concept of neighborhood of a vertex. Let V be the vertex set of *Triangulene oxide network*. An r – *neighborhood* of a vertex v is is defined by ,

$$N_r(v) = \{u \in V : d(u,v) = r\}$$

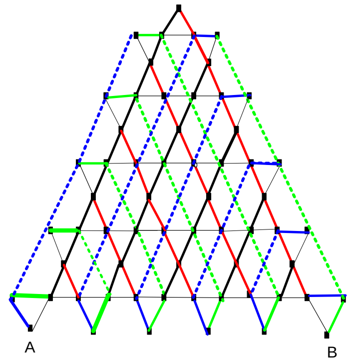

Fig. 17. Neighborhood of a vertex A and vertex B

Note:

1) $N_r(A)$ are vertices connected by green and red line segments.
2) $N_r(B)$ are vertices connected by dark black and blue line segments.

We denote by P_x, a segment of an X-line consisting of points (x, y), with x coordinate fixed.

That is line segments

P_x = { (x_0, y) / $y_1 \le y \le y_2$}, $x > 0$

Similarly,

P_y = { (x, y_0) / $x_1 \le x \le x_2$}, $y > 0$
$P_{x=0}$ = { (0, 2n-1)}
$P_{y=0}$ = { (2n-1,0)}

Lemma 1: In any Regular Triangulene Oxide network, for $1 \le r \le 2n+1$.

- For r is even and less than $2n$.
  1) $N_r(A) = P_{x=r-1}$ - { (r-1,2n), (r-1, 2n-r-1)}
  2) $N_{2n}(A) = P_{x=2n-1}$ - { (r-1, 2n-r-1)}
  3) $N_r(B) = P_{y=r-1}$ - { (2n,r-1), (2n-r-1, r-1)}
  4) $N_{2n}(B) = P_{y=2n-1}$ - { (2n-r-1, r-1)}
  5) $N_1(A) = P_{x=0}$ - { (r-2,2n)}
  6) $N_1(B) = P_{y=0}$ - { (2n, r-2)}
- For r is odd and less than $2n+1$
  1) $N_r(A) = P_{x=r-1} \cup$ { (r-2,2n), (r-2, 2n-r)}
  2) $N_{2n+1}(A) = P_{x=2n} \cup$ { (r-2, 2n-r)}
  3) $N_r(B) = P_{y=r-1} \cup$ { (2n,r-2), (2n-r, r-2)}
  4) $N_{2n+1}(B) = P_{y=2n} \cup$ { (2n-r, r-2)}.

Proof: For $1 \le r \le 2n-1$ and x_i (x_i = -1, 1, 3, 5…, 2n-3)is odd and less than 2n-1,

$$P_{x=x_i} = \{ (x_i, 2n-s) \ / \ s = 0,1,2,3\ldots, x_i+2 \}$$
$$P_{x=2n-1} = \{ (x_i, 2n-s) \ / \ s=1,2,3\ldots, 2n+1 \}$$

For x_i is even (x_i =0,2,4,…,2n-2)

$$P_{x=x_i} = \{ (x_i, 2n-1-t)/t = 0,2,4\ldots x_i \}$$
$$P_{x=2n} = \{ (x_i, 2n-1-t) \ / \ t = 2,4,6,\ldots, 2n \}$$
$$P_{y=y_i} = \{ (2n-s, y_i) \ / \ s = 0,1,2,3\ldots, y_i+2 \}$$
$$P_{y=2n-1} = \{ 2n-s , y_i) \ / \ s = 1,2,3\ldots, 2n+1 \}$$

For y_i is even (y_i =0, 2, 4,…, 2n-2)

$$P_{y=y_i} = \{ (2n-1-t, y_i)/t = 0,2,4,\ldots y_i \}$$

$P_{y=2n} = \{ 2n\text{-}1\text{-}t , y_i) / t=2,4,6,\ldots, 2n \}$,

Hence the lemma1.

Lemma 2: For any r_1 and r_2, $N_{r_1}(A) \cap N_{r_2}(B)$ is either empty or singleton set.

Proof: Suppose the theorem is wrong, that is there exist two distinct vertices $u(x_1, y_1)$ and $v(x_2, y_2)$ such that u and v belongs to $N_{r_1}(A) \cap N_{r_2}(B)$, implies $u(x_1, y_1)$, $v(x_2, y_2)$ belong to $N_{r_1}(A)$ and $N_{r_2}(B)$. Without loss of generality, let us assume that r_1, r_2 are even numbers. Now $u(x_1, y_1)$, $v(x_2, y_2)$ belong to $N_{r_1}(A)$ implies, clearly u and v lies in $P_{x=} r_1\text{-}1$, and therefore the " x" - co ordinates of u and v are r_1-1. *That* is $x_1= r_1$-1, and $x_2= r_1$-1. Then u and v can be represented by

$$u(r_1\text{-}1, y_1),\ v(r_1\text{-}1, y_2) \qquad (1)$$

Now $u(x_1, y_1)$, $v(x_2, y_2)$ belongs to $N_{r_2}(B)$ implies, clearly u and v lies in $P_{y=}r_2\text{-}1$, and therefore the "y"- co ordinates of u and v are r_2-1.

That is $y_1= r_2$-1, and $y_2= r_2$-1. Then u and v can be represented by

$$u(x_1, r_2\text{-}1),\ v(x_2 , r_2\text{-}1) \qquad (2)$$

From the equation (1), (2) and unique representation of u and v implies $x_1= x_2$ and $y_1= y_2$, implies $u =v$, which is a contradiction to initial assumption that u and v are distinct. Hence lemma 2.

Corollary3: Let $u=(x_1,y_1)$, $v=(x_2,y_2)$ be vertices of Regular Triangulene Oxide network , $x_1\neq x_2$, $y_1\neq y_2$ then $N_{r_1}(A) \cap N_{r_2}(B)$ contains at most one of u and v.

Proof: Suppose both u and v belongs to $N_{r_1}(A) \cap N_{r_2}(B)$, then it is contradiction to lemma2. Now we shall prove

{A, B} is a metric basis for Regular Trianglulene Oxide network.

Case1:

If u and v lies in $N_{r_1}(A)$ (3)

and $d(x ,y)$ denotes the distance between x and y.

then $d(u , A)$=d(v , A) . Let us prove $d(u , B)\neq d(v , B)$

Suppose $d(u , B)$=$d(v , B)$ then there exist $N_{r_2}(B)$ such that ,

u and v lies in $N_{r_1}(B)$ (4)

from equation (3) and (4)

u and v belongs to $N_{r_1}(A) \cap N_{r_2}(B)$, which is a contradiction to corollery3. Thus $d(u , B)\neq d(v , B)$.

Case 2:

Similarly we can prove $d(u , A)\neq d(v , A)$ if u and v lies in $N_{r_1}(B)$.

Case 3:

If $u \in N_{r_1}(A), and\ v \in N_{r_2}(B)$, it is clear that d(u , A)≠ d(v , A). Similarly we can show $d(u , B)\neq d(v , B)$ if $u \in N_{r_1}(B),\ v \in N_{r_2}(A)$.

Case 4:

If $u \in N_{r_1}(A), and\ v \in N_{r_2}(B)$ then there exist $N_{r_3}(A)$ such that $v \in N_{r_3}(A)$ [Since the vertex set V is the disjoint union of $N_r(A)$, r=1, 2,…,2n+1]. Thus u and v are as in case 3.Hence {A, B} is metric basis for Regular Triangulene Oxide network. Thus ,We have proved the following theorem.

Theorem 4.1: Minimum metric dimension of *Regular Triangulene Oxide network* of dimension n is 2.

V. Conclusion

In this paper, We have proved that the Minimum metric dimension of *Regular Triangulene Oxide network* is 2. Research work has been continued to derive new architecture from *Dominating Silicate network* to study various topological properties and solve problems like Minimum metric dimension of Networks and Embedding of Networks in to regular Networks.

Symbol Index

$\gamma(G)$: Number of nodes(vertices) in a graph G.

$e(G)$: Number of edges in a graph G.

K_3: Complete graph on 3 nodes.

$\lceil\beta\rceil$: Roof of β, i.e. the smallest integer greater than or equal to β.

$\lfloor\alpha\rfloor$: Floor of α, i.e. the largest integer less than or Equal to α.

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