Masking Matrix Design of Quasi-Cyclic LDPC Codes for High-speed Optical Communications

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Abstract—**This paper deals with a masking matrix design method of algebraically constructed quasi-cyclic (QC) LDPC codes for high-speed optical communications. The structure of masking matrix of QC-LDPC codes influences not only code rate, but also cycle and trapping sets distributions, so that it decides the code performance to a great extent, especially in error floor region. With the proposed method, masking matrix is carefully designed to have dual-diagonal and cyclic-shift structure, so as to improve implementation complexity. At the same time, the code rate can be flexibly adjusted and low storage memory is required. Simulation results show that the designed codes with high rate have excellent performance and error floor does not appear until bit error rate 10^{-8}.**

Index Terms—**High-speed, masking matrix, optical communications, quasi-cyclic LDPC codes.**

I. Introduction

Modern fiber-optic communication systems have evolved at a rapid space. Demands for higher transport speed and higher spectral efficiencies inspire the applications of digital signal processing, new modulation and coding technique in optical communication systems. Among them, forward error correction (FEC) codes play a very important role to compact various kinds of impairments.

FEC codes applied in optical communication systems have developed from Reed-Solomon (RS) code, concatenated BCH codes or RS codes, to iteratively decodable codes such as low density parity check (LDPC) codes and the net coding gain (NCG) has improved from 5.6dB, around 8dB to more than 10dB [1]-[2]. Recently, many attentions have been paid to works on new efficiency FEC codes such as product code or LDPC codes.

Among different kinds of LDPC codes, algebraically constructed quasi-cyclic (QC) LDPC codes [3]-[5] is especially suitable for high speed optical communication systems benefiting from the low implementation complexity and its excellent error floor performance. The construction process of algebraically constructed QC-LDPC codes includes the designs of base matrix, masking matrix and matrix expansion to achieve the corresponding parity check matrix. The design of base matrix and matrix expansion for QC-LDPC codes have been extensively studied in [5], however the design of good masking matrix is still an open problem, even though the structure of masking matrix influences not only the code rate, but also cycle and trapping sets distributions, so that it decides the code performance to a great extent, especially in error floor region.

In this paper, a new masking matrix design method of QC-LDPC codes is presented with which the designed QC-LDPC codes give very low error floor performance with high code rate so that they are suitable for high-speed optical communication systems. Simulation results illustrate that NCG can achieve 10.13dB with code rate 0.9375 and code length 32640 at bit error rate (BER) 10^{-8}.

II. Preliminaries

An LDPC code can be characterized by its parity check matrix $\boldsymbol{H}$ with the i-th row and j-th column element $h_{i,j}$. The construction process of LDPC codes can be looked on as the design process of the corresponding parity check matrix $\boldsymbol{H}$. The encoding and decoding implementation can both be done with the parity check matrix $\boldsymbol{H}$. For algebraically constructed QC-LDPC code, the construction of its parity check matrix $\boldsymbol{H}$ can be realized in three steps: the base matrix design, masking matrix design and matrix expansion to get the required parity check matrix $\boldsymbol{H}$. In the followings, these three steps are represented separately.

A. Base Matrix

Base matrix $\boldsymbol{B}$ can be denoted as

$$B=\begin{bmatrix} b_{0,0} & b_{0,1} & \cdots & b_{0,\rho-1} \\ b_{1,0} & b_{1,1} & \cdots & b_{1,\rho-1} \\ \vdots & \vdots & \ddots & \vdots \\ b_{\gamma-1,0} & b_{\gamma-1,1} & \cdots & b_{\gamma-1,\rho-1} \end{bmatrix}. \quad (1)$$

It has γ rows and ρ columns and each element $b_{i,j}$ $i\in\{0,1,\ldots,\gamma\text{-}1\}$, $j\in\{0,1,\ldots,\rho\text{-}1\}$, comes from finite field GF(q^m) with q being a prime number and m a positive integer. All the elements of finite field GF(q^m) can be denoted as $\{0, \alpha^0{=}1, \alpha^1, \alpha^2,\ldots, \alpha^{q^m-2}\}$ where α denotes a primitive element of GF(q^m).

Studies already show that, short cycles especially length-4 cycles have greatly influence on the code performance. To ensure the designed LDPC codes have no length-4 cycles, α-multiplied RD-constrain must be satisfied for base matrix $\boldsymbol{B}$. The detailed design rules and corresponding proofs can be referenced in [5].

B. Masking Matrix

To obtain a sparse matrix as a foundation to generate sparse parity check matrix of QC-LDPC codes, some non-zero elements of base matrix $\boldsymbol{B}$ should be replaced by

Manuscript received September 4, 2012; revised October 8, 2012.

This work was supported in part by NSFC (Nos. 61101148 and 61001130), the Fundamental Research Funds for the Central Universities (No. K50510010006), and the 973 Program (No. 2010CB328300).

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DOI: 10.7763/IJFCC.2013.V2.171

zero elements, which can be realized by masking. Masking matrix $\boldsymbol{Z}$ has the same number of rows and columns as base matrix $\boldsymbol{B}$, and it can be represented as follows:

$$Z = \begin{bmatrix} z_{0,0} & z_{0,1} & \cdots & z_{0,\rho-1} \\ z_{1,0} & z_{1,1} & \cdots & z_{1,\rho-1} \\ \vdots & \vdots & \ddots & \vdots \\ z_{\gamma-1,0} & z_{\gamma-1,1} & \cdots & z_{\gamma-1,\rho-1} \end{bmatrix}, \quad (2)$$

where the value of $z_{i,j}$ $i\in\{0,1,\ldots,\gamma\text{-}1\}$, $j\in\{0,1,\ldots,\rho\text{-}1\}$, only can be 0 or 1. Then taking the following matrix product and getting matrix $\boldsymbol{M}$:

$$\boldsymbol{M}=\boldsymbol{Z}\times\boldsymbol{B}=[\, z_{i,j}\times b_{i,j}], \quad (3)$$

where $z_{i,j}\times b_{i,j}= b_{i,j}$ with $z_{i,j}$=1 and $z_{i,j}\times b_{i,j}= 0$ with $z_{i,j}$=0. After above operation, it can be seen that the achieved matrix $\boldsymbol{M}$ is sparser than the original base matrix $\boldsymbol{B}$.

Actually by above masking operation, two advantages are obtained. On one side, masking results in a sparser matrix whose associated Tanner graph has fewer edges and hence fewer short cycles and probably a larger girth than that of the associated Tanner graph of the original base matrix $\boldsymbol{B}$. On the other side, masking results in the change of row or column weight of the parity check matrix, so it can be used to adjust the row or column weight and code rate of the required codes.

C. Matrix Expansion

Finite field GF(q^m) totally has q^m elements$\{0, \alpha^0=1, \alpha^1, \alpha^2,\ldots, \alpha^{q^m-2}\}$ among which one is zero and the others q^m-1 elements is non-zero. If the following mapping is assumed:

$$\alpha^t \rightarrow W\left(\alpha^t\right), t=0,1,\ldots,q^m-2, \quad (4)$$

where $\boldsymbol{W}(\alpha^j)$ is a (q^m-1)× (q^m-1) matrix with the following elements:

$$w_{i,j} = \begin{cases} 1, & i\in\left\{0,1,\ldots,q^m-2\right\}, j=\left(i+t\right)\%\left(q^m-1\right) \\ 0, & others \end{cases}, \quad (5)$$

where $w_{i,j}$ denotes the i-th row and j-th column element of matrix $\boldsymbol{W}(\alpha^j)$ and % denotes model 2 operation.

By (4), non-zero elements in GF(q^m) can be mapped to (q^m-1)× (q^m-1) matrices one by one and the zero element in GF(q^m) can be mapped to (q^m-1)× (q^m-1) all-zero matrix.

Apply above mapping to every element of matrix $\boldsymbol{M}$, and then get the matrix $\boldsymbol{H}$ whose elements are 0 or 1. The matrix $\boldsymbol{H}$ is the parity check matrix of designed QC- LDPC codes. The code length of designed codes is $\rho(q^m$-1).

III. Masking Matrix Design

The main contribution of this paper is presented in this part that a masking matrix design method is proposed. With this method, the designed QC-LDPC codes not only have the dual-diagonal parity check matrix which facilitates the encoding implementation in a linear time complexity, but also present excellent code performance for high rate codes especially in error-floor region which is exactly suitable for high speed optical communication systems.

A. Masking Matrix Design Process

The generating process of proposed masking matrix $\boldsymbol{Z}$ is described as follows:

Step 1: set the masking matrix $\boldsymbol{Z}$ being the dual-diagonal structure:

$$Z = \begin{bmatrix} z_{0,0} & z_{0,1} & \cdots & z_{0,\rho-\gamma-1} & 1 & 1 & \cdots & 0 \\ z_{1,0} & z_{1,1} & \cdots & z_{1,\rho-\gamma-1} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ z_{\gamma-1,0} & z_{\gamma-1,1} & \cdots & z_{\gamma-1,\rho-\gamma-1} & 1 & 0 & \cdots & 1 \end{bmatrix}. \quad (6)$$

Step 2: let $a=\rho/\gamma$(Assume that ρ can be divided byγ.), then $z_{i,j}$, $i\in\{0,1,\ldots,\gamma\text{-}1\}$,$j\in\{0,1,\ldots,\rho\text{-}\gamma\text{-}1\}$ can be divided into a-1 $\gamma\times\gamma$square matrix Z_t, $t\in\{0,1,\ldots, a\text{-}2\}$:

$$Z_t = \begin{bmatrix} z_{0,t\gamma+0} & z_{0,t\gamma+1} & \cdots & z_{0,t\gamma+\gamma-1} \\ z_{1,t\gamma+0} & z_{1,t\gamma+1} & \cdots & z_{1,t\gamma+\gamma-1} \\ \vdots & \vdots & \ddots & \vdots \\ z_{\gamma-1,t\gamma+0} & z_{\gamma-1,t\gamma+1} & \cdots & z_{\gamma-1,t\gamma+\gamma-1} \end{bmatrix}. \quad (7)$$

For every Z_t, $t\in\{0,1,\ldots, a\text{-}2\}$, randomly choose b positions from $z_{0,\, t\gamma+j}$, $j\in\{0, 1,\ldots, \gamma\text{-}1\}$ and set the values of these positions 1 and the values of the other positions 0.

Step 3: set the i-th $i\in\{1,\ldots, \gamma\text{-}1\}$ row of Z_t, $t\in\{0,1,\ldots, a\text{-}2\}$ as follows:

$$z_{i,\, t\gamma+j}= z_{i\text{-}1,\, t\gamma+j\text{-}1}, j\in\{1, 2,\ldots, \gamma\text{-}1\} \text{ and } z_{i,\, t\gamma}= z_{i\text{-}1,\, t\gamma+\gamma\text{-}1}. \quad (8)$$

In other words, the elements of every row except the first row are right cyclic shift of its previous row.

Step 4: set the i-th (i is odd and $0<i<\gamma$) row of Z_t, $t\in\{0,1,\ldots, a\text{-}2\}$ as follows:

$$z_{i,\, t\gamma+j}=(z_{i\text{-}1,\, t\gamma+j}+1)\%2, j\in\{0, 1,\ldots, \gamma\text{-}1\}. \quad (9)$$

After above four steps, the obtained masking matrix $\boldsymbol{Z}$ is just what we wanted.

Masking matrix can determine the row and column weight of designed QC-LDPC codes. From above process, it can be known that the row weight is (a-1)b+2 or (a-1)(γ-b)+2 and column weight is γ/2 or 2.

B. Characteristics of Designed Masking Matrix

1) The dual-diagonal structure of above masking matrix can lead the designed parity check matrix to dual-diagonal structure which allows the encoding process can be implemented in linear time complexity.
2) The number of zeros in masking matrix can be adjusted by parameter b, which determines the decreased number of the edges in associated Tanner graph and further the number of short cycles that influences the code performance.
3) The designed LDPC code is irregular and the corresponding degree distributions of variable nodes or parity check nodes in associated Tanner graph can be adjusted by parameters γ and b.

IV. Simulation Results

The performance of algebraically constructed QC-LDPC codes with the proposed masking matrix is simulated by computer in this part.

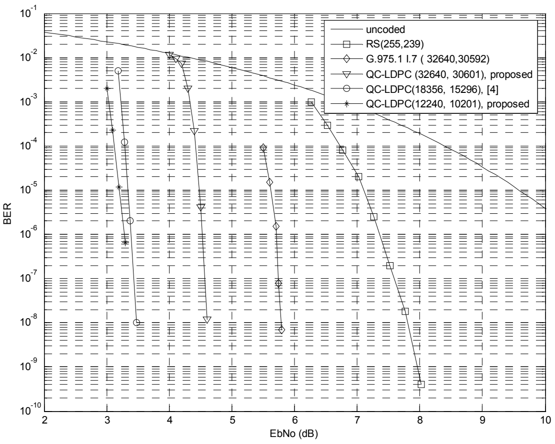

Fig. 1. Simulation results of QC-LDPC codes with proposed masking matrix design algorithm and comparisons with existed codes.

A. Simulation Environment

In the simulation of this paper, additive white Gaussian noise (AWGN) channel is assumed to approximate high speed optical channel, that is reasonable for the working region of signal-to-noise ratio in optical communications.

Generally for the researchers of forward error correction, the code rate *R* and signal-to-noise ratio EbNo (dB) are used as a figure of merit of binary modulation schemes. However in optical communications, it is a common practice to use redundancy instead of code rate and the Q-factor (dB) instead of EbNo. The relationships between them are: redundancy refers to as $(n\text{-}k)/k$ and code rate as n/k; Q= EbNo+$10\times\log_{10}(2\times R)$, where n denotes code length and k denotes the number of information bits in a codeword. In our simulation, parameters R and EbNo are used and the corresponding redundancy and Q-factor can be obtained with above relationships between them.

In our simulation, the adopted base matrix of QC-LDPC codes is in [5]. Masking matrix is designed with our proposed algorithm. Matrix expansion is done with the depiction in part II.C. The obtained QC-LDPC codes are simulated by sum-product decoding algorithm with iterative number 50 and the iterative decoding process stops when 100 error frames occur or the total tested frames reach 10^7. Assume all-zero codeword is transmitted and BER is made as a statistic merit in the Monte Carlo simulation process.

The adopted code rate is carefully chosen for high speed optical communications. Current long-haul fiber-optic communication systems use FEC codes with 7% redundancy (code rate 0.9375), as recommended in ITU-T G.975 and its successor ITU-T G.975.1 [6]. As the speed of electronics increased, it became practical to consider more powerful FEC codes so that more redundancy is permitted now. The new FEC overhead (OH) limit is 20%, which is suggested by the OIF in [7]. Therefore, two code rates, 0.9375 and 0.833, are adopted in the following simulation results.

B. Simulation Results

Fig. 1 illustrates simulation results of our proposed QC-LDPC codes and their comparisons with the existed FEC schemes designed for high speed optical communication systems.

Two simulation results of QC-LDPC codes with our proposed masking matrix design algorithm are presented in Fig. 1. The code rates are 0.9375 and 0.833 separately.

The first code is QC-LDPC (32640, 30601), the curve marked with triangles. The parameters of its masking matrix are γ=8, ρ=128, a=16, b=4 and base matrix comes from GF(2^8). This code is compared with two orthogonally concatenated BCH super FEC code given in G.975.1 I.7 section [6], the curve marked with diamonds in Fig. 1. It can be seen that our proposed code improves about 1.2dB in EbNo (3.93dB in Q-factor) than two orthogonally concatenated BCH super FEC code with the same code length 32640 and code rate 0.9375. NCG is 10.13dB in Q-factor at BER 10^{-8}. The performances of RS (255, 239) code and un-coded system are also referenced in Fig. 1.

The second code is QC-LDPC (12240, 10201), the curve marked with circles, whose masking matrix is set to γ=8, ρ=48, a=6, b=4. Compared to the QC-LDPC code in [4], which is the first report about FPGA-based verification of soft-LDPC codes proving transmission without error-floor down to BER of 10^{-15}, about 0.1dB performance improvement is achieved for our designed code with the same code rate 0.833. It is worth noting that the code length of our code is smaller than that of the code in [4], which is the reason that the slope of our code performance curve is a little smaller than that of the code in [4].

V. Conclusions

As the rapid development of optical transmission systems and the speed of electronics, advanced FEC codes are desirable and practical for high speed optical communication systems. In this paper, a new masking matrix design method is proposed for construction of QC-LDPC codes applied in optical communication systems. Simulation results show that the designed QC-LDPC codes present excellent performance.

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