Adaptive Segmentation Gaussian Mixtures Models for Approximating to Drastically Scaled-Various Sloped Long-Tail RTN Distributions

Worawit Somha and Hiroyuki Yamauchi

***Abstract*—This paper proposes a fitting method to approximate the mixtures of various sloped-tail Gamma distribution characterizing the random telegraph noises (RTN) by an adaptive segmentation Gaussian mixtures model (GMM). The concepts central to the proposed method are 1) adaptive segmentation of the long-heavy tailed distributions such that the log-likelihood of GMM in each partition is maximized and 2) copy and paste with an adequate weight into each partition. This allows the fitting model to apply various bounded tail distribution even with multiple convex and concave folding curves. It is verified that the proposed method can reduce the error of the fail-bit predictions by 2-orders of magnitude while reducing the iterations for EM step convergence to 1/16 at the interest point of the fail probability of 10^{-12} which corresponds to the design point to realize a 99.9% yield of 1Gbit chips.**

***Index Terms*—Mixtures of Gaussian, random telegraph noise, em algorithm, heavy-tail distribution, long-tail distribution, fail-bit analysis, static random access memory, guard band design.**

I. Introduction

The approximation-error of the tails of random telegraph noise (RTN) distribution will become an unprecedentedly crucial challenge resulting from the fact that: (1) its error directly leads to the error of the guard band (GB) design required to avoid the out of spec after shipped to the market, and (2) tails of RTN distribution will become much longer than that of random-dopant-fluctuation (RDF) which is the conventional dominant factor of the whole margin-variations and the convolution results of the two will be more affected by the RTN than the RDF, as can be seen in Fig. 1. Since the increasing paces of variation-amplitude Vth are differently dependent on the MOSFET channel-size (LW) like the below expressions of (1) and (2), the Vth increasing paces of RTN is a 1.4x faster than that of RDF if assuming the LW is scaled down to 0.5 every process generation, as shown in Fig. 1(a).

$$\Delta \mathrm{Vth}\ (\mathrm{RDF}) \propto \mathrm{AVt}\ (\mathrm{RDF}) \ / \sqrt{\mathrm{LW}} \quad (1)$$

$$\Delta \mathrm{Vth}\ (\mathrm{RTN}) \propto \mathrm{AVt}\ (\mathrm{RTN}) \ / \ \mathrm{LW} \quad (2)$$

where AVt (RDF) and AVt (RTN) are Pelgrom coefficients for RDF and RTN, respectively.

Manuscript received December 15, 2012; revised February 23, 2013. This work was supported in part by MEXT/JSPS KAKENHI Grant Number of 23560424 and grant from Information Sceience Laboratory of Fukuoka Institute of Technology.

The authors are with the Information Intelligent System Fukuoka Institute of Technology, 3-30-1, Wajiro-Higashi, Higashi-ku, Fukuoka, Japan (e-mail: bd12002@ bene.fit.ac.jp, yamauchi@fit.ac.jp).

This means that RTN will soon exceed RDF and becomes a dominant factor of whole margin variations, as shown in Fig. 1(a). According to the references [1]-[5], there will come the time soon around 15nm scaled CMOS era.

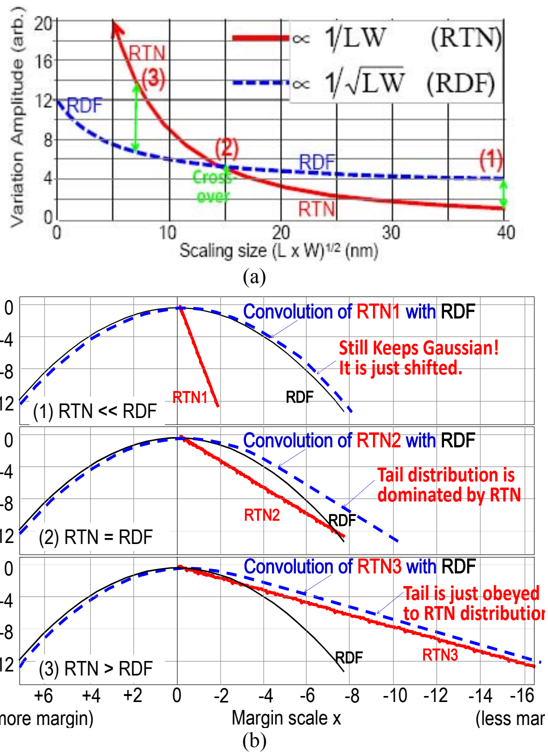


Fig. 1. (a) Trend of variation amplitude of RTN and RDF (b) Comparisons of distributions of convolution results between 3 cases of assuming RTN for 40nm, 16nm, and 7nm class device scaling. (1) RTN<RDF, (2) RTN=RDF, (3) RTN>RDF. RTN will dominate the whole variations.

To make clear the issues we will discuss in this paper, the concepts of what will happen at that time are shown in Figs. 1-2. Fig. 1(b) illustrates the probability density functions for RDF, RTN1(40nm), RTN2(<16nm) and RTN3 (<7nm), and its convolution results, respectively.

It is worth mentioning that the distribution-shape of the convolution results obey the Gaussian when RTN<RDF and changes to follow the combinations of Gamma and Gaussian distributions when RTN=RDF, and finally becomes dominated by Gamma distribution of RTN when RTN>RDF, respectively [4]-[5]. The tails on the both sides of the distribution are asymmetrical and are differently influenced by longer-tail Gamma-RTN for right side and shorter tail Gaussian-RDF for left side and, respectively, as shown in Fig. 1(b).

Since the interest area for the GB design is on the right side, i.e., in less margin zone, it can be seen that the approximation-error of the RTN distribution directly leads to estimation-error of fail-bit counts (FBC). The conventional

DOI: 10.7763/IJFCC.2013.V2.195

Gaussian-model characterizing for the whole-margin variation can't be used any more for analyzing such non Gaussian long-tail distributions of RTN. Fig. 2 shows how the affects of the approximation-error on the FBC error will be increased as the process dimension is scaled down. Until 15nm, its impact can be increased by 6 orders of magnitude compared to that of 40nm, as shown in Fig. 2.

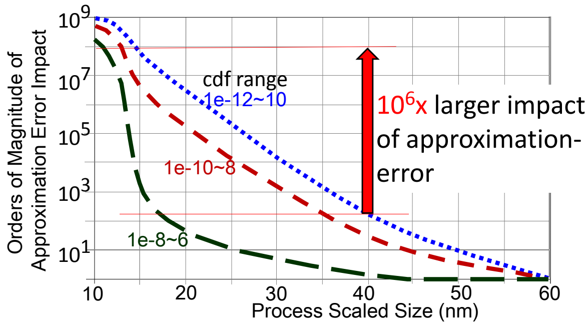


Fig. 2. Increased impact of approximation-error on the trouble of excessive under-estimation/over- estimation of the fail-bit counts.

In order to solve the above issues, we propose, for the first time, a fitting method to approximate a long-tailed RTN distribution by an adaptive segmentation Gaussian mixtures model (GMM). This provides the following benefits: 1) applicable to the various convex and concave shapes of bounded Gamma distribution even with the wide range of shape-parameter =0.02 to 1.15 and 2) still using Gaussian distribution to simply utilize an error-function for cumulative density function. The main contribution of this paper is to point out that it is possible to approximate the long tailed distributions by mixtures of convenient Gaussian probability distributions, so that available yield-prediction models can be effectively analyzed and so that the effect of the long tailed distributions upon the fail-bit count accuracy can be analytically determined. This is because the convolution result of linear combinations of Gaussians becomes also Gaussians and can be expressed by the analytical expressions, which allows using normal (Gaussian) cumulative density function (normcdf) for estimating the error counts. This makes it easier to predict the fail-bit counts before and after screening at the stages of both circuit design and screening test [1]-[3].

Here is how the rest of this paper is organized. In Section II, we refer to some of the example as evidence indicating how the conventional models cause intolerable huge error to make clear the purpose of the proposed work. In Section III, we will propose our recursive algorithm for constructing approximating Gaussian mixtures model (GMM). In Section IV, we refer to some of the example as evidence indicating if the proposed models can approximate well the heavy long-tailed distributions. We give a precise fail-bit count prediction. In Section V, we rigorously prove that it is possible to approximate various long-tailed distributions with bounded convex and concave curves by mixtures of Gaussian distributions. Finally, we state our conclusion in Section VI.

II. Discussions on the Conventional Models

Expectation-maximization (EM) algorithm [6], which is an iterative procedure that maximizes the likelihood of Gaussian mixtures models (GMM), is well known as easy and convenient means to approximate GMM to the non Gaussian distributions.

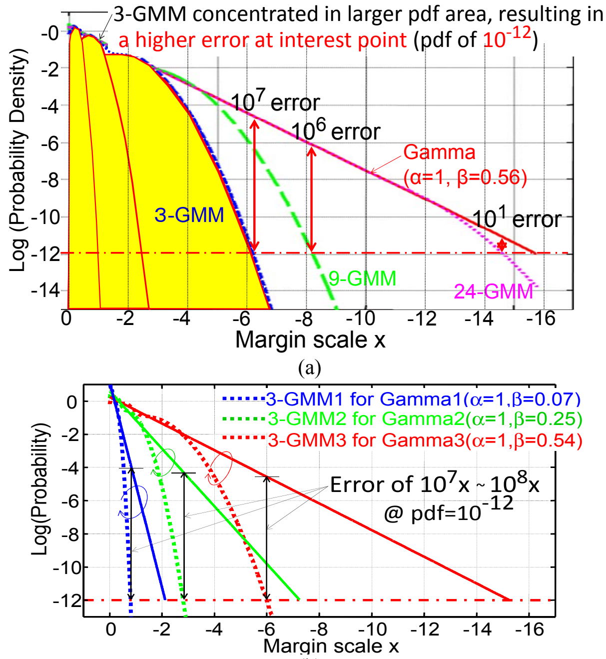


Fig. 3. (a) Approximation error comparisons between 3, 9, 24-GMMs cases: errors of orders of 10^7 , 10^6, and 10^1, respectively. (b) Error dependency of 3 types of Gamma distributions of =0.07, 0.25 and 0.54, respectively.

However, all GMMs given by this fitting algorithm tend to concentrate in the non-tail region in which the sensitivity to increase the likelihood is much larger than that for the tail region, as shown in Fig. 3. Since the interest region for analyzing the fail-bit counts of the rare-events is in the tail region (at probability of 10^{-12}), the EM algorithm for this application leads to a significant fail-bit count error of orders of 10^7, as shown in Fig. 3. Even if increasing the number of GMM from 3 to 9 and 24, the significant error of orders of 10^6 and 10^1, respectively, are still remained, as shown in Fig. 3. In almost all fail-bit analyses, the distribution of interest only matters in the tail-region of probability of orders of 10^{-12} [1]-[3]. Thus, this is a crucial challenge we should address until the time comes for the rare-event SRAM yield predictions.

III. Proposed Statistical Approximation Model for RTN Gamma Distribution

In order to solve these crucial issues, we develop a remarkably simple adaptively segmentation EM algorithm-based fitting algorithm. The centerpiece of this idea is: (a) adaptive partitioning of the long tailed distributions such that the log-likelihood of GMM is maximized in each segmentation and (b) copy and paste fashion with an adequate weight into each partition for constructing the whole long-tail distributions. The concepts of the two different proposed EM-based approximation means are shown in Fig. 4(a) and (b), respectively.

A. Adaptive Segmentation

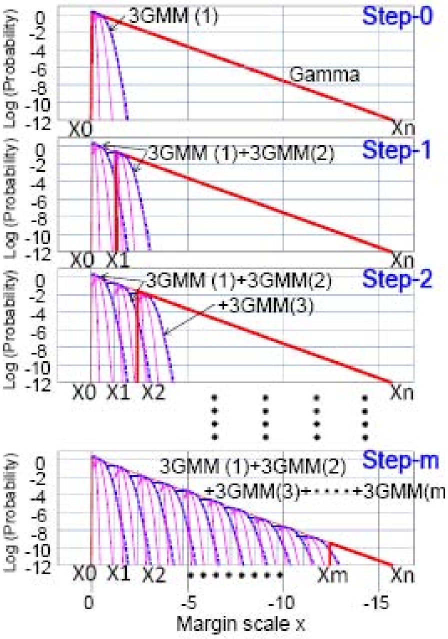
Algorithm of the adaptive segmentation is described below from step 1) to step 4).

1) 1st-step is to do approximation by 3-GMM between X0 and Xn. And find the point of X1, where likelihood of 3-GMM is maximized.

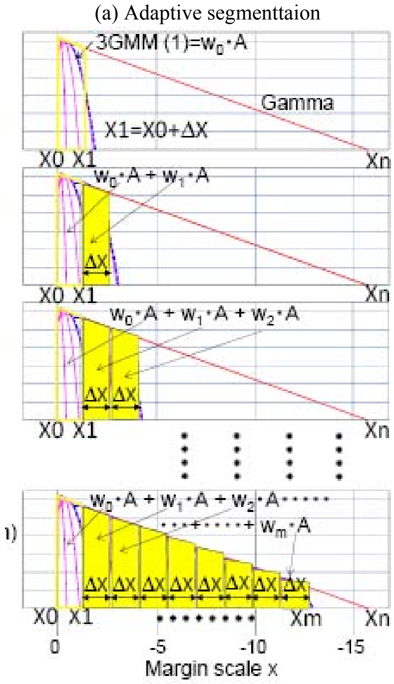
2) 2[nd]-step is to do the same thing as 1) between X1 and Xn. And find the point of X2, where likelihood of 3-GMM is maximized.

3) 3[rd]-step is to do the same thing as 2) between X2 and Xn. And find the point of X3, where likelihood of 3-GMM is maximized between X3 and Xn.

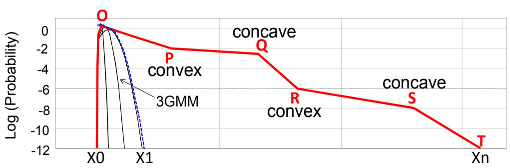
This flow can be repeated until the likelihood of whole GMM can be maximized as shown in Fig. 4(a).



(a) Adaptive segmenttaion



(b) Copy and paste fashion



(c) Example of complex distributions comprising various variation factors

Fig. 4. Concepts of the proposed approximation algorithm. (a) adaptive segmenttaion:Xm are decided such that likelifood of each 3-GMM can be maximized. (b) copy and paste fashion: copy the first 3-GMM and paste to others with adaptive weighting. (c) example of complex distributions comprising various variation factors

B. *Copy and paste fashion*

Algorithm of the copy and paste fashion is described below from step 1) to step 4).

1) 1[st]-step is to do approximation by 3-GMM between X0 and Xn. And find the point of X1, where likelihood of 3-GMM is maximized. X is given by (X1-X0) and w_0 is the weight of the 1[st] 3-GMM.

2) 2[nd]-step is to get the weight (w_1) of the 2[nd] 3-GMM. And copy the 1[st] 3-GMM and paste it into the adjacent place (shifted by X) by weighting of w_1, which is given by the slope of Gamma distribution.

   where, slope=(w_0 - w_1)/ X

3) 3[rd]-step is to do the same thing as 2), as shown in Fig. 4(b). This flow can be repeated until Xm>Xn.

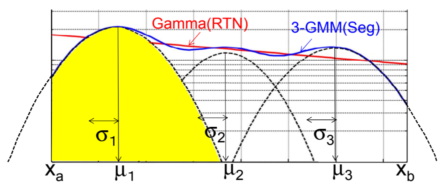
This algorithm can allow approximating any heavy-long tailed distributions by the convenient short-tail Gaussian probability distributions. Even if the whole distributions are comprised of mixtures of various convex and concave curves as shown in Fig. 4(c), individual area of (O-P), (P-Q), (Q-R), (R-S), and (S-T) can be adaptively segmented based on its slope. It is a clear that the both proposed ideas can apply to this kind of distribution. In Section V, an example of actual distributions of future RTN is given and discussed.

Thanks to the segmentation, the range of variables for the 3-GMM approximation is limited and almost similar to the other segmentations. This can make the number of EM-iterations required to find the best point smaller and help to avoid the wrong convergence point unlike the conventional EM-algorithm, as shown in Table I. This also allows us to use only Gaussian distributions when doing convolution of Gaussian-RDF and Non-Gaussian-RTN distributions.

TABLE I: COMAPRISONS OF EM-ITERATIONS

	Segmentation (proposed)	Conventional (w/o segmentation)
Gamma1	3	338
Gamma2	6	340
Gamma3	6	340

The convolution results also can be given by analytical simple and convenient expressions of just linear combination of Gaussian, which can give us the fail-bit count by just summing up the values of normal (Gaussian) cumulative density function (normcdf) for each Gaussian of the whole GMM. The example of how to caluculate the fail-bit error counts of the segmentation of (x_a-x_b) is shown in Fig. 5.



S_1=normcdf(x_b,μ_1,σ_1)-normcdf(x_a,μ_1,σ_1)
S_2=normcdf(x_b,μ_2,σ_2)-normcdf(x_a,μ_2,σ_2)
S_3=normcdf(x_b,μ_3,σ_3)-normcdf(x_a,μ_3,σ_3)

Fig. 5. Error bit counts of the segmentation of (x_a-x_b) can be given by just summing up the normal (Gaussian) cumulative density function (normcdf) of three GMMs.

IV. Discussion on Accuracy of Statistical Approximation Model for RTN Distribution

To illustrate the effects of the proposed scheme on the reduction of the approximation-error in the long tails, the following examples are assumed: (1) ratio of how fast does the tail decay of Gaussian-RDF and Gamma-RTN, i.e., its parameters are assumed as follows: (σ=1, μ=0) for Gaussian, (α=1, β=0.56) for Gamma. The relationship between the two distributions and its convolution are shown in Fig. 1b) and (2) comparisons of the following 6 approximation-models of Gamma distribution (α=1, =0.56): (a) the number of 3, 9, 24, and 128 of GMMs are used for fitting the whole distribution (no segmentation) and (b) the number of 3 and 9 of GMMs are used for approximating each segmentation comprising whole distribution.

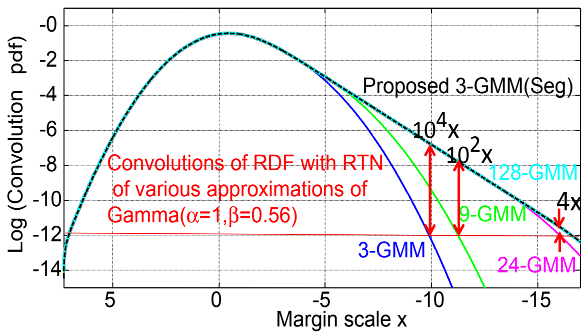


Fig. 6. Comparisons of the convolution results between the cases with and without segmentation schemes of 3, 9, 24, and 128-GMMs. Proposed one can reduce the error by 10^4x, 10^2x, and 4x than 3, 9, and 24-GMMs, respectively

Fig. 6 shows the comparisons of the convolution results between the cases with and without segmentation schemes. It is found that 3-GMM segmentation scheme can reduce the errors by 7-orders and 4-orders of magnitude at the fail probability of 10^{12}, as shown in Fig. 6. It is worth mentioning that 3-GMM segmentation scheme provides a better approximation than the case of 24-GMM, as shown in Fig. 6 In order to characterize the error of each convolution result, the "golden", which is given by the numerical calculation of convolution of Gaussian and Gamma distributions, is used as a reference. The numbers of fail-bit count errors for each approximation model are compared, as shown in Fig. 7. The number of fail-bit count error is defined as the difference in the cdf value between the "golden" and that for each model.

It is worth mentioning that the relationship of "which is better" is dependent on the x-scale, as shown in Fig. 7. For example, 128-GMM is the best in X= (-6~-12) in which the fail-probability is larger than 10^{-8} (shown in Fig. 7). In contrast, the proposed one can reduce the error best in X= (-12~-16) in which the fail-probability is smaller than 10^{-8} where there is the most interest point for the GB designs.

When discussing the GB designs for volume production, the expected yield-loss should be predicted. We assumed the target here that fail-probability is 10^{-12} to realize 99.9% yield of 1Gbit memory chip, which is a quite realistic target. Thus, our most interest point of x is around x=-16, where fail-probability is around 10^{-12}, as you can see by Fig. 7. In that sense, we can say that our proposed 3-GMM segmentation method can provide the best approximation compared with others, as shown in Fig. 7.

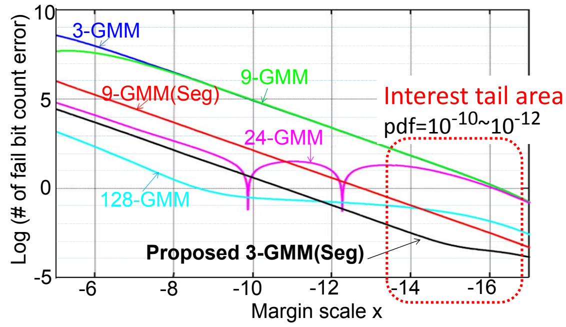


Fig. 7. Comparisons of the numbers of fail-bit count errors between 6 approximation means of 3, 9, 24, and 128 GMM without segmentation and 3 and 9-GMM with segmentation schmes.

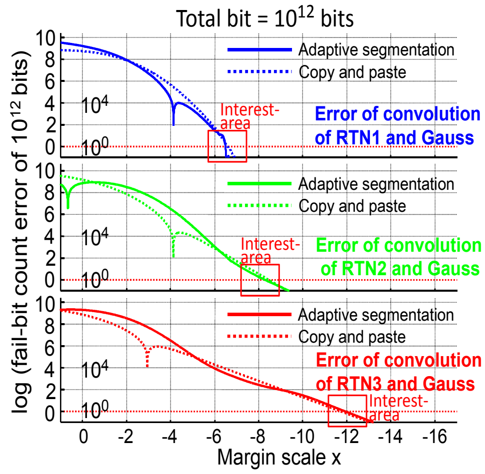


Fig. 8. Comparisons of the numbers of fail-bit count errors between the adaptive segmentation and copy and paste fashion for the 3-cases of convolution of RTN1, RTN2, and RTN3. Total # of bits are assumed as 10^{12}.

As mentioned earlier, 9-GMM segmentation is worth than 3-GMM segmentation in the wide range of x=-6~-16 because the variation of probability becomes larger and the density of GMM in area of lower probability becomes much smaller as shown in Fig. 7. Regarding the cases without using segmentation, the number of errors is 1~4 orders of magnitude larger than that for the proposed 3-GMM segmentation scheme.

Fig. 8 shows the comparisons of the numbers of fail-bit count errors between the adaptive segmentation and copy and paste fashion we proposed in this paper. The 3-cases of the errors of the convolution of Gauss with RTN1, RTN2, and RTN3 are shown, respectively. Although the small difference in terms of the fail-bit counts can be seen in the non-interest area, it is found that the both ideas of the proposed adaptive segmentation and copy and paste fashion can provide the small enough accuracy in the interest area (rare-event), compared with the conventional means, as shown in Figs 7-8.

V. Application to More Complex Distributions

According to the reference [7]- [9], the distribution of RTN amplitude will have a complex bounded tail caused by "atomistic" variation-behaviors with various variation factors of the gate line-edge roughness (GER), fin-edge roughness (FER), and metal gate granularity (MGG), as shown in Fig. 9. They are no longer obeyed to the single gamma distribution but to the multiple gamma distribution depending on the tail positions of (O-P), (P-Q), and (Q-R), as shown in Fig. 9. As the examples to illustrate the effectiveness of the proposed fitting models, the three types of distributions whose have a different folding points are given as Combo1, Combo2, and Combo3, as shown in Fig. 9.

The proposed both ideas of "adaptive segmentation" and "copy and paste" fashion can apply to this kind of complex non-linear distribution. This is because the width of each segmentation is much smaller than the length of (O-P), (P-Q), and (Q-R). The same concepts can be used in each partition of (O-P), (P-Q), and (Q-R).

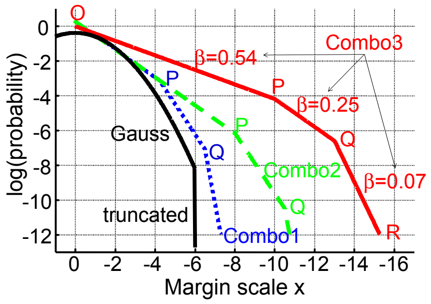


Fig. 9. Various bounded tails of the distributions of Gauss (RDF) and combination of different shaped gamma distributions of Combo1, Combo2, and Combo3.

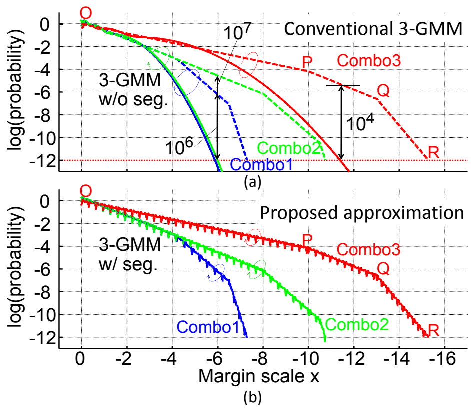


Fig. 10. Comparisons of errors for fitting to Combo1, Combo2, and Combo3 between the cases of (a) with the convnetional 3-GMM model and (b) with the proposed segmentation models.

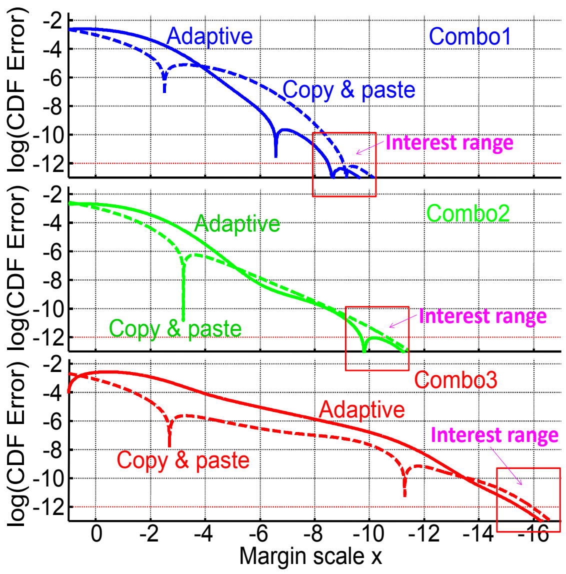


Fig. 11. Comparisons of the errors of cumulative density function (cdf) of the convolution results for Combo1, Combo2, and Combo3 between the case of using the "adaptive segmentation" and "copy and paste" fashion.

Fig. 10 shows the comparisons of approximation-errors for fitting to Combo1, Combo2, and Combo3 between the cases of (a) using the convnetional 3-GMM model and (b) using the proposed segmentation models. As can be seen in the Fig. 10(a), the conventional 3-GMM models without using segmentation manner can't fit the tails of Combo1-3 at all. The errors of 4,6, and 7 orders of maginitude have to be expected at the rare probability of 10^{-12}. Contrary, the fitting errors can be drastically reduced by using the proposed ideas, as shown in Fig. 10(b). Unlike the case of Fig. 10(a), it can be seen that the fitting curves and its target lines in Fig. 10(b) are perfectly overlapped. Thanks to the segmentation manner, the same concepts can be adaptively applied to the different sloped-tail distributions. This indicates that this ideas can be applied to the various sloped-distributions even if they are combined like the given examples in Fig. 9.

Since the both ideas of "adaptive segmentation" and "copy and paste" fashion can apply to this kind of complex non-linear distribution, the errors of cumulative density function (cdf) of the convolution results for Combo1, Combo2, and Combo3 are compared between the two, as shown in Fig. 11.

It is found that the trend of cdf errors depending on the margin scale of x position is similar between the different distributions of Combo1-3, as can be seen in Fig. 11.
The cdf errors for the "copy and paste" are smaller than that for the "adaptive segmentation" in the smaller x-position. Contrary, its relationship is inverted. Since the region of a larger x and a smaller probability like 10^{-12} is more interest area for the rare event fail-bit count analyses, it can be said that the proposed idea of "adaptive segmentation" provides the better fitting model to predict the yield-loss after shipped to the market due to the time-dependent RTN-caused failures.

VI. Conclusion

In this paper, we have discussed, for the first time, how the various-sloped RTN distribution-tail should be approximated and how much its approximation-error can affect on the accuracy of the statistical predictions of the number of fail-bit counts, which is required to avoid the out of spec after shipped to the market. It has been pointed out that the conventional Gaussian models can't be used any more due to intolerable model errors caused by the deviation from the actual RTN-caused distributions, once the distribution-tail of the RTN becomes longer than that of the conventional variations of the RDF. This is because the tail of convolution results doesn't obey to the Gaussian any more but follows to the mixtures of various-sloped Gamma distributions.

To address the above issues, we have proposed the two types of an effective simple algorithm for approximating the tails of RTN distributions by convenient and simple GMM. This allows the fitting model to apply the various bonded tail distributions even with the multiple convex and concave folding curves. It has been verified that the proposed method can reduce the error of the fail-bit predictions by 2-orders of magnitude while reducing the iterations for EM step convergence to 1/16 at the interest point of the fail probability of 10^{-12} which corresponds to the design point to realize a 99.9% yield of 1Gbit chips.

We have also pointed out that the proposed scheme is a candidate fitting algorithm for the distributions of the future RTN distributions, which will be crucial not only for the circuit design but also the GB design for screening test when RTN variables becomes larger than that of RDF.

ACKNOWLEDGMENT

The authors are grateful to Yan Zhang, Yu Ma for their helps.

REFERENCES

[1] H. Yamauchi, "A discussion on SRAM circuit design trend in deeper nanometer-scale technologies," *Very Large Scale Integration (VLSI) Systems, IEEE Transactions on* Vol. 18, Issue :5, pp. 763-774, 2010.

[2] H. Yamauchi, "Embedded SRAM trend in nano-scale CMOS," Memory Technology, Design and Testing, MTDT 2007. *IEEE International Workshop2007 on*, pp. 19-22, 2007.

[3] H. Yamauchi, tutorials "Variation tolerant SRAM circuit design" *IEEE ISSCC 2009 and IEEE A-SSCC 2008,* 2008.

[4] K. Takeuchi, et al, "Comprehensive SRAM Design Methodology for RTN Reliability" , in *Digest of IEEE Symposium on VLSI Technology 2011*, pp. 130-131, 2011.

[5] K. Takeuchi, et al, "Direct Observation of RTN-induced SRAM Failure by Accelerated Testing and Its Application to Product Reliability Assessment " , in *Digest of IEEE Symposium on VLSI Technology 2010*, pp. 189-190, 2010.

[6] Moon, T.K, "The expectation-maximization algorithm" *Signal Processing Magazine, IEEE*, Volume: 13, Issue: 6, pp. 47-60, 1996.

[7] X. Wang, et al, "RTS amplitude distribution in 20nm SOI FinFETs subject to Statistical Variability," *Simulation of Semiconductor Processes and Devices SISPAD 2012*, pp.296-299, 2012.

[8] X. Wang, et al, "Simulation Study of Dominant Statistical Variability Sources in 32-nm High-k/Metal Gate CMOS," *IEEE Electron Device Letters - IEEE ELECTRON DEV LETT* , vol. 33, no. 5, pp. 643-645, 2012.

[9] K.P.Cheung, et al," The amplitude of random telegraph noise: Scaling implications," *Reliability Physics Symposium (IRPS), 2012 IEEE International*, pp.1.1-1.3

Worawit Somha received master degree in electrical engineering from King Mongkut's Institute of Technology Ladkrabang (KMITL), Bangkok, Thailand. His master thesis was on "Vector Quantizers for Speech Coding and there Implementation on TMS-320C30". Since 1995 he has given a lecture for bachelor degree student in subject of "Introduction to Digital Signal Processing" at KMITL as an assistance professor, and his research area is speech coding. Since 1997 he has worked with the company in the position of consulting engineering.

Since 2012 he has got the scholarship from KMITL for D.Eng. student program and now being pursuing PhD degree in major of intelligence information system engineering at Fukuoka Institute of Technology.

Hiroyuki Yamauchi received the Ph.D. degree in engineering from Kyushu University, Fukuoka, Japan, in 1997. His doctoral dissertation was on "Low Power Technologies for Battery-Operated Semiconductor Random Access Memories". In 1985 he joined the Semiconductor Research Center, Panasonic, Osaka, Japan. From 1985 to 1987 he had worked on the research of the submicron MOS FET model-parameter extraction for the circuit simulation and the research of the sensitivity of the scaled sense amplifier for ultrahigh-density DRAM's which was presented at the 1989 Symposium on VLSI Circuits. From 1988 to 1994, he was engaged in research and development of 16-Mb CMOS DRAM's including the battery-operated high-speed 16 Mbit CMOS DRAM and the ultra low-power, three times longer, self-refresh DRAM which were presented at the 1993 and 1995 ISSCC, respectively. He also presented the charge-recycling bus architecture and low-voltage operated high-speed VLSI's, including 0.5V/100MHz-operated SRAM and Gate-Over-Driving CMOS architecture, which were presented at the Symposium on VLSI Circuits in 1994 and 1996, respectively, and at the 1997 ISSCC as well. After experienced general manager for development of various embedded memories, eSRAM, eDRAM, eFlash, eFeRAM, and eReRAM for system LSI in Panasonic, he has moved to Fukuoka Institute of Technology and become a professor since 2005. His current interests are focused on study for variation tolerant memory circuit designs for nano-meter era. He holds 212 Patents including 87 U.S. Patents and has presented over 70 journal papers and proceedings of international conferences including 10 for ISSCC and 11 for Symposium on VLSI Circuits. Dr. Yamauchi received the 1996 Remarkable Invention Award from Science and Technology Agency of Japanese government and the highest ISOCC2008 Best Paper Award.

He had been serving a program commitee of ISSCC for long periods, from 2002 through 2009.

He served a program committee of IEEE Symposium on VLSI Circuits from 1998 through 2000 and has come back and been serving again since 2008. He is also serving A-SSCC since 2008.