Comparison of Direction of Arrival (DOA) Estimation Techniques for Closely Spaced Targets

Nauman Anwar Baig and Mohammad Bilal Malik

Abstract—**This paper deals with different beamforming techniques for DOA estimation. High resolution techniques such as Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) which are subspace based techniques are also discussed. Finally, further resolution improvement is achieved through the use of directional sensors. The computational complexities of beamforming techniques are also compared.**

Index Terms—**Beamforming, computations, direction-of-arrival, multiple, radar, resolution.**

I. Introduction

Beamforming uses an array of receivers (sensors) for directional transmission/reception. The main goal for formation of an array and array processing is to combine the sensors outputs so that the SNR can be enhanced, information about the number of sources/targets and direction of each can be determined, various parameters of the incident signals can be estimated.

The main requirement in many source localization applications e.g. Radar, Sonar is to estimate the direction of arrival without errors. When two sources have a small angular distance between them in space, angular resolution is an important area to concern about; otherwise some sources/targets would not be detected. The objective of paper is to discuss DOA estimation algorithms with focus on the enhancement of angular resolution. The requirement is achieve a higher resolution with minimum number of computations.

Propagating signals contain much information about the sources that produce them. Not only does each signal's waveform express the nature of the source, its temporal and spatial characteristics combined with the laws of physics allow us to determine the source's location [1]. For propagating signals, more is needed; spatiotemporal filtering must be employed to separate signals according to their directions of propagation and their frequency content.

II. Model for Incident Waves

A uniform linear array (ULA) is used for beamforming. The sources/targets are assumed to be in the far-field region so that the waves coming from them to the ULA can be considered to be plane waves. The data from the ULA is

$$\mathbf{x}(n) = \mathbf{A}(\phi)\mathbf{s}(n) + \mathbf{n}(n) \qquad (1)$$

Here, $\mathbf{A}(\phi)$ $(N \times M)$ is the steering matrix where N is the number of sensors and M is the number of sources. $\mathbf{s}(n)$ is the signals vector and $\mathbf{n}(n)$ is additive white Gaussian noise [2]. The number of snapshots of the signals are K, n=1, 2, 3,....,K.

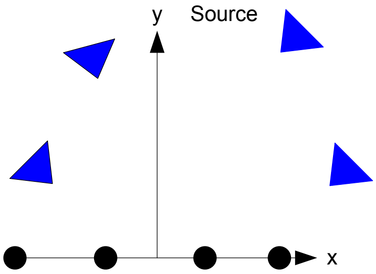

Fig. 1. Multiple signals incident on ULA

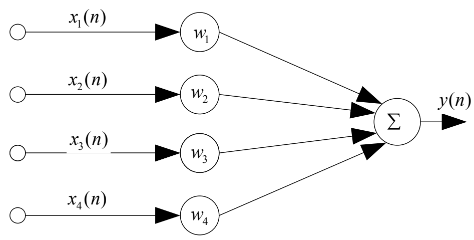

Fig. 1. Beamformer

It is also assumed that the signal and noise both are zero mean, noise variance at each of the sensors is σ^2. The sample correlation matrix is given by

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{n=1}^{K}\mathbf{x}(n)\mathbf{x}^H(n) \qquad (N \times N) \qquad (2)$$

III. Beamforming Techniques

Beamforming techniques can be divided into spectral estimation techniques and subspace based methods [3]. In spectral estimation, a spectrum-like function of the parameter of interest e.g. the DOA is formed. The locations of the highest (separated) peaks of the function are the DOA estimates. The idea is to steer the array in one direction at a time and measure the output power. The steering locations which result in maximum power yield the DOA estimates.

The beamformer's output power is [3]

$$P(\mathbf{w}) = \mathbf{w}^H\hat{\mathbf{R}}\mathbf{w} \qquad (3)$$

A. Conventional Delay and Sum Beamformer (CBF)

It maximizes the power of the beamformer output for a given input signal. For CBF, weight vector is the steering

Manuscript received October 29, 2012; revised February 15, 2013.
The authors are with Department of Electrical Engineering, College of E&ME, National University of Sciences and Technology, Islamabad, Pakistan (e-mail: nau_baig@yahoo.com, mbmalik67@yahoo.com)

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vector. For different angles, the output power is measured [3]

$$P(\phi) = \mathbf{a}^H(\phi)\hat{\mathbf{R}}\mathbf{a}(\phi) \tag{4}$$

where the steering vector

$$\mathbf{a}(\phi) = g(\phi)\left[1\ e^{-jkd\sin\phi}\ \ e^{-jk2d\sin\phi} \ldots\ldots\ e^{-jk(N-1)d\sin\phi}\right]^T \tag{5}$$

And $g(\phi) = 1$, for omnidirectional sensors. Following plot shows the CBF output power when 2 sources are present at $\phi_1 = -30^o$ and $\phi_1 = 20^o$

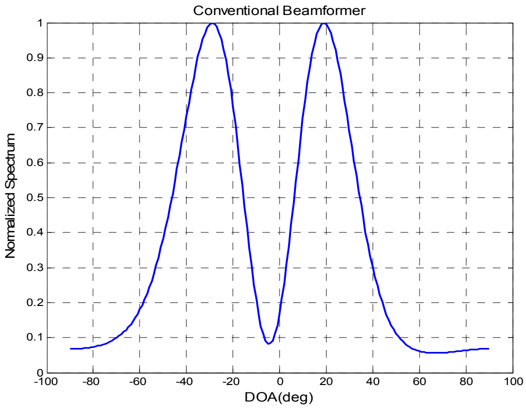


Fig. 3. N=4 Sensors, SNR=10dB and K=200 samples=200

A limitation of the CBF is that it cannot resolve 2 targets within the beamwidth. Consider 2 sources at 0° and 20°

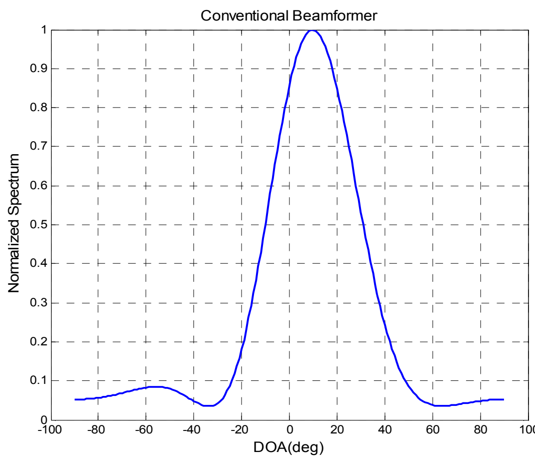


Fig. 4. Can't resolve targets inside beamwidth of 4 sensors i.e. 30°

One way to increase the resolution is to increase the number of sensors, which reduces the beamwidth. But it also increases the cost of the beamformer.

B. Minimum Variance Distortionless Response Beamformer (MVDR)

To reduce the limitations of conventional beamformer, such as to increase the resolving power of two sources spaced closer than a beamwidth, a method was proposed by Capon [4]. The power spectrum is

$$P(\phi) = \frac{1}{\mathbf{a}^H(\phi)\hat{\mathbf{R}}^{-1}\mathbf{a}(\phi)} \tag{6}$$

Following plot shows the comparison of resolutions of CBF and MVDR

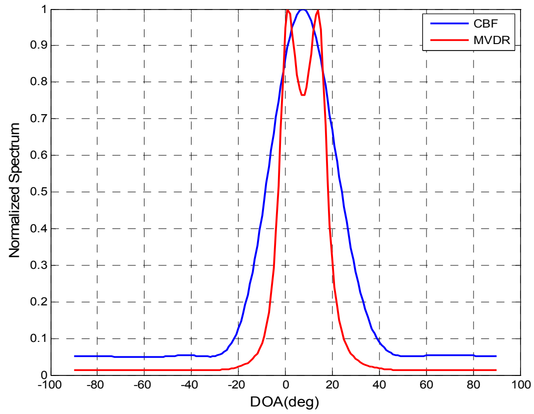


Fig. 5. N=4, Sources at 0° and 15°

From the above figure, it can be seen that the resolution of MVDR is much better than that of CBF.

Now the high resolution subspace based methods are discussed which are based on the decomposition of correlation matrix into *signal* and *noise* subspaces.

$$\mathbf{R} = \mathbf{APA}^H + \sigma^2\mathbf{I} = \underbrace{\mathbf{U}_s\Lambda_s\mathbf{U}_s^H}_{signal\ subspace} + \underbrace{\mathbf{U}_n\Lambda_n\mathbf{U}_n^H}_{noise\ subspace} \tag{7}$$

C. Multiple Signal Classification (MUSIC)

This algorithm uses the fact that all noise eigenvectors are orthogonal to the signal steering vectors [5]

$$\mathbf{U}_n^H\mathbf{a}(\phi) = 0, \quad \phi \in \{\phi_1, \phi_2, \ldots, \phi_M\} \tag{8}$$

The output spectrum for MUSIC beamformer is

$$P(\phi) = \frac{1}{\left|\mathbf{U}_n^H\mathbf{a}(\phi)\right|^2} = \frac{1}{\mathbf{a}^H(\phi)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\phi)} \tag{9}$$

In the output spectrum, M largest peaks correspond to DOAs

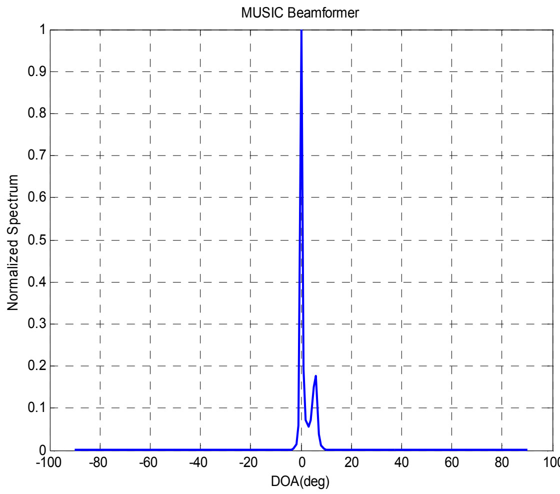


Fig. 6. N=4, 2 sources separated by 5°

The above plot shows that using 4 sensors, even two closely spaced targets can be resolved using MUSIC beamformer. When the number of snapshots (available data is small), a modified version of MUSIC known as Root-MUSIC proves to be useful. It is based on polynomial rooting.

D. Root-MUSIC

The Root -MUSIC method converts the MUSIC spectrum into a polynomial whose solution results directly in numeric values for the estimated directions [6].

E. Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT)

This algorithm is based on translational invariance structure (e.g.ULA) of sensors [7]. ESPRIT is computationally more efficient as it doesn't require an exhaustive search through all possible steering vectors for DOA estimation. Following figure shows the pair of subarrays used in ESPRIT

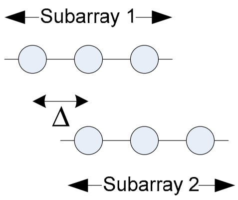


Fig. 7. Two identical subarrays

The DOA estimates are given by

$$\phi_k = \sin^{-1}\left[\frac{\arg(\Phi_k)}{\beta d}\right] \tag{10}$$

where ϕ_k is the estimated DOA, Φ_k is the k^{th} eigen value of subspace rotational operator [7], $\beta = 2\pi/\lambda(wavelength)$ and d is the spacing between the sensors. The following plot shows the output of ESPRIT beamformer when sources were present at -2°, 1° and 4°.

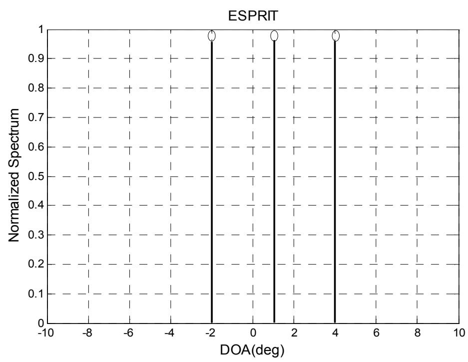


Fig. 8. High resolution ESPRIT beamformer

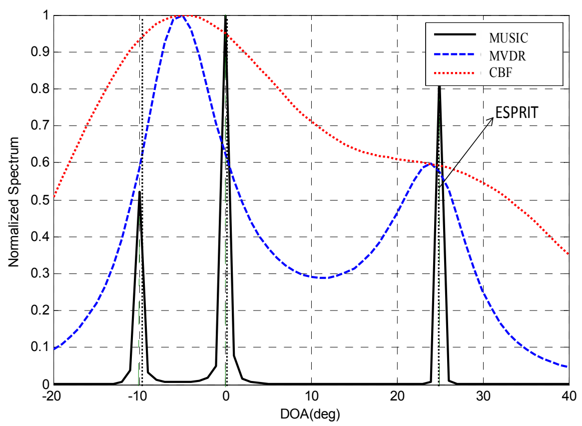


Fig. 9. Resolution comparison of BEAMFORMERS

IV. Comparison of Beamforming Algorithms

Following figure shows the resolving power of different beamformers discussed

From the above plot, it can be seen that the ESPRIT is a very high resolution algorithm.

In the following plot, the accuracy of beamformers for different SNR values are compared. The RMSE is given by

$$RMSE = \frac{\sqrt{E(\phi_1 - \hat{\phi}_1)^2} + \sqrt{E(\phi_2 - \hat{\phi}_2)^2}}{2} \tag{11}$$

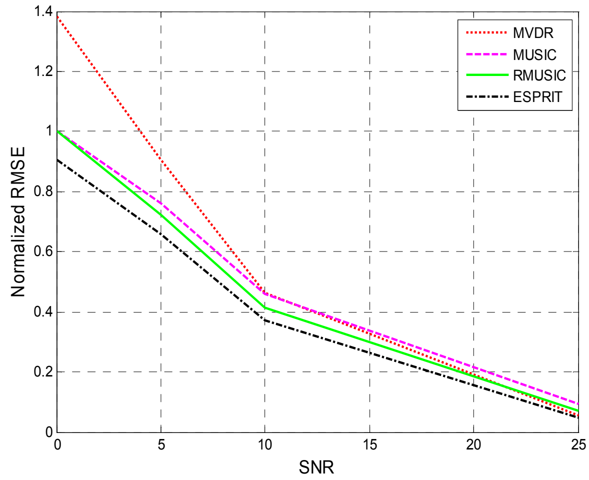


Fig. 10. N=4, separation=10°, 100 trials and 200 samples

It can be observed that ESPRIT is the most accurate for low SNR conditions.

TABLE I: Summary of Beamforming Algorithms

Algorithm	Resolution	Complexity	General Remarks
CBF	Poor	Simple Implementation, 1-D search	Resolution depends on main lobe
MVDR	Good	Inverse of **R**, 1-D search	Poor performance in low SNR
MUSIC	Very Good	Eigenvalue Decomposition, 1-D search	Also estimates number of sources [5]
ESPRIT	Excellent	Eigenvalue Decomposition, Calculating $\boldsymbol{\psi}$	Array needs doublets

V. Computational Complexities

The following tables show the number of multiplications and additions required for each of the algorithm. The following symbols are used (Table I-Table VI)

N = number of sensors

M = number of signals

K = number of samples

L = number of angles to scan

TABLE I: COMPUTATION OF CORRELATION MATRIX

Operation	Multiplications	Additions	Divisions
$\mathbf{R}_{N\times N}=\frac{1}{K}\sum_{n=1}^{K}\mathbf{x}(n)_{N\times 1}\mathbf{x}^H(n)_{1\times N}$	$K\left(\frac{N^2}{2}+\frac{N}{2}\right)$	$(K-1)N^2$	N^2

TABLE II: COMPUTATIONS OF CBF

Operation	Multiplications	Additions	Divisions
$P(\phi)_{1\times 1}=\mathbf{a}^H(\phi)_{1\times N}\mathbf{R}_{N\times N}\mathbf{a}(\phi)_{N\times 1}$	$L(N^2+N)$	$L(N^2-1)$	–

TABLE III: COMPUTATIONS OF MVDR

Operation	Multiplications	Additions	Divisions
$\mathbf{R}^{-1}_{N\times N}$ (Gauss Jordan Inversion) [8]	$\frac{N^3}{3}+\frac{N^2}{2}+\frac{N}{6}$	$\frac{N^3}{3}+\frac{N^2}{2}+\frac{N}{6}$	$\frac{N^2}{2}+\frac{N}{2}$
$P(\phi)_{1\times 1}=\frac{1}{\mathbf{a}^H(\phi)_{1\times N}\hat{\mathbf{R}}^{-1}_{N\times N}\mathbf{a}(\phi)_{N\times 1}}$	$L(N^2+N)$	$L(N^2-1)$	L

TABLE IV: COMPUTATIONS OF MUSIC ALGORITHM

Operation	Multiplications	Additions	Divisions
$\mathbf{R}_{N\times N}=\mathbf{U}_{N\times N}\mathbf{\Lambda}_{N\times N}\mathbf{U}^H{}_{N\times N}$ [9], [10]	$\frac{16}{5}N^3$	$\frac{44}{5}N^3$	–
$\mathbf{Q}_{N\times N}=\mathbf{U}_{n[N\times(N-M)]}\mathbf{U}^H_{n\,[(N-M)\times N]}$	$\frac{(N-M)N^2}{2}+\frac{(N-M)N}{2}$	$\frac{(N-M)N^2}{2}-\frac{N^2}{2}+\frac{(N-M)N}{2}-\frac{N}{2}$	–
$P(\phi)_{1\times 1}=\frac{1}{\mathbf{a}^H(\phi)_{1\times N}\mathbf{Q}_{N\times N}\mathbf{a}(\phi)_{N\times 1}}$	$L(N^2+N)$	$L(N^2-1)$	L

TABLE V: COMPUTATIONS OF ESPRIT ALGORITHM

Operation	Multiplications	Additions	
$\mathbf{R}_{N\times N}=\mathbf{U}_{N\times N}\mathbf{\Lambda}_{N\times N}\mathbf{U}^H{}_{N\times N}$	$\frac{16}{5}N^3$	$\frac{44}{5}N^3$	
$\mathbf{A}_{M\times M}=\mathbf{U}^H_{s1[M\times(N-1)]}\mathbf{U}_{s1[(N-1)\times M]}$	$\frac{(N-M)(N-1)^2}{2}+\frac{(N-M)(N-1)}{2}$	$\frac{(N-M)(N-1)^2}{2}-\frac{(N-1)^2}{2}+\frac{(N-M)(N-1)}{2}-\frac{(N-1)}{2}$	
			Divisions
$\mathbf{B}_{M\times M}=\mathbf{A}^{-1}_{M\times M}$ (Gauss Jordan Inversion) [8]	$\frac{N^3}{3}+\frac{N^2}{2}+\frac{N}{6}$	$\frac{N^3}{3}+\frac{N^2}{2}+\frac{N}{6}$	$\frac{N^2}{2}+\frac{N}{2}$
$\mathbf{C}_{M\times M}=\mathbf{U}^H_{s1[M\times(N-1)]}\mathbf{U}_{s2[(N-1)\times M]}$	$\frac{(N-M)^2(N-1)}{2}+\frac{(N-M)(N-1)}{2}$	$\frac{(N-M)^2(N-1)}{2}-\frac{(N-M)^2}{2}+\frac{(N-M)(N-1)}{2}-\frac{(N-M)}{2}$	
$\mathbf{\Psi}_{M\times M}=\mathbf{B}_{M\times M}\mathbf{C}_{M\times M}$	$(N-M)^3$	$(N-M)^3-(N-M)^2$	

VI. ENHANCEMENT IN RESOLUTION USING DIRECTIONAL SENSORS

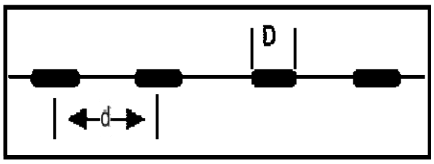
When directional sensors are used instead of omnidirectional sensors, the half power beamwidth of the array's response reduces which increases the resolution of the beamformer.

Consider an array of 4 sensors each having a linear aperture of length D ($D<d$)

The array patterns are compared below where 4 sensors are used

TABLE VI: FOR FIXED N, M, K AND L

Computational complexity decreases downward
MVDR
CBF
MUSIC
ESPRIT



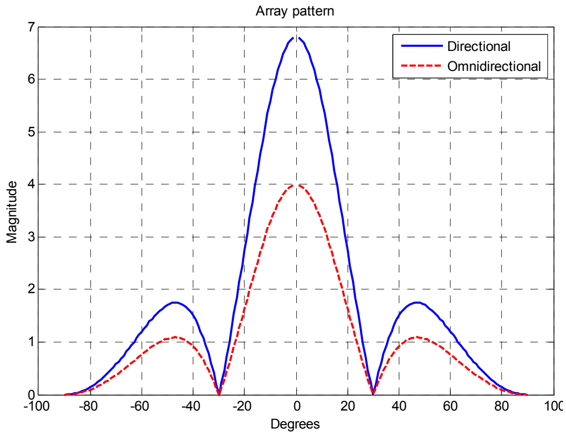


Fig. 11. Array patterns of directional and omnidirectional sensor array

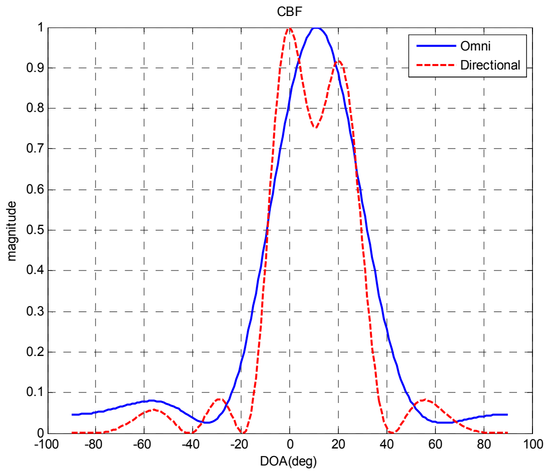


Fig. 12. CBF for directional array is better

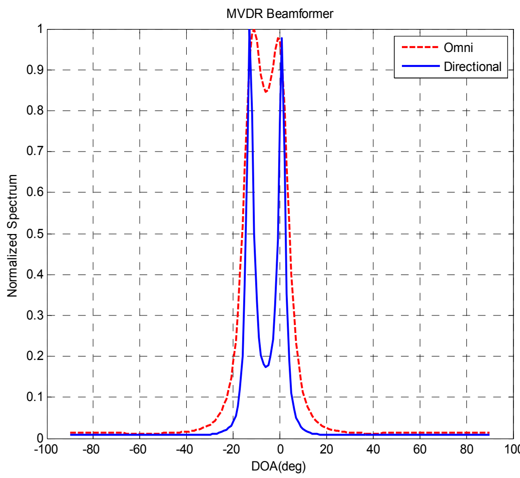


Fig. 13. Response of 4 directional sensors

The output of CBF is shown below. The output power of each of the beamformers with directional and omnidirectional sensors is first normalized and then combined

Similarly for MVDR and MUSIC beamformer, the increase in resolution obtained by using directional sensors is shown below

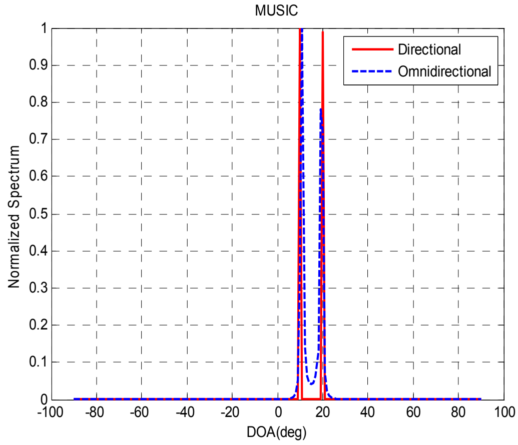


Fig. 14. MUSIC beamformer

VII. CONCLUSIONS

In this paper different algorithms for estimation of direction of arrival are discussed. The main focus is to increase the resolution so that closely spaced targets in space can be separated. The conventional beamformer has a resolution limitation due to beamwidth. It has been shown that beamwidth can be reduced by increasing the number of sensors but it also inceases the cost of beamformer. Adaptive beamforming algorithms have the advantage of much better resolution. The minimum variance distortionless response beamformer has a relatively higher resolution due to the output power minimization subject to the constraint.

Subspace methods for estimation of DOA are based on the signal and noise subspaces. MUSIC algorithm which shows high peaks for angles corresponding to DOAs, has a much higher resolution. It has also been shown that using directional sensors instead of omnidirectional sensors gives the advantages of a relatively reduced beamwidth and higher gain. The tables of computational complexities show that ESPRIT is computationally much efficient as it does not require a scan through all possible angles.

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Nauman Anwar Baig received his B.S. degree in electrical engineering from University of Engineering & Technology, Taxila, Pakistan, in 2008. He received his M.S. degree in electrical engineering from College of Electrical and Mechanical Engineering (E&ME), Rawalpindi, Pakistan, in 2010. Currently he is doing Ph.D. in Electrical Engineering from College of Electrical and Mechanical Engineering (E&ME), National University of Sciences and Technology (NUST), Rawalpindi, Pakistan. His research focuses on signal processing and communications.

Mohammad Bilal Malik received his B.S. degree in electrical engineering from College of Electrical and Mechanical Engineering (E&ME), Rawalpindi, Pakistan, in 1991. He received his M.S. degree in Electrical Engineering from Michigan State University (MSU), Michigan, USA, in 2001. In 2004, he received his Ph.D. degree in Electrical Engineering from National University of Sciences and Technology (NUST), Rawalpindi, Pakistan. He has been teaching at College of E&ME, National University of Sciences and Technology (NUST) since 1991. His research focuses on signal processing and adaptive filtering for communications and control systems.