

# A Robust Convertible Multi-Authenticated Encryption Scheme with One-Way Hash Function

Chung-Fu Lu, Chien-Lung Hsu, Han-Yu Lin, and Chien-Hui Su

**Abstract**—A convertible multi-authenticated encryption (CMAE) scheme allows a group of signers to cooperatively produce a valid authenticated encryption signature and still preserves the characteristic of convertible authenticated encryption (CAE) schemes. In 2008, Tsai proposed a CMAE scheme based on the intractability of one-way hash functions and discrete logarithms. However, we find that Tsai's scheme cannot provide the semantic security of the message and the computational efficiency of this scheme is rather high. In this paper, we propose a secure CMAE scheme, in which it enhances the semantic security of the message to overcome the defect in Tsai's scheme. Moreover, by compared with Tsai's scheme, the performance of the proposed scheme is more efficient than Tsai's in term of computational complexity.

**Index Terms**—Semantic security, authenticated encryption, one-way hash function.

## I. INTRODUCTION

Elaborating on the Nyberg-Rueppel message recovery signature scheme [1], Horster *et al.*, proposed an authenticated encryption (AE) scheme [2]. An AE scheme allows one signer to generate an authenticated ciphertext such that only a designated recipient has the ability to decrypt the signed message and verify its corresponding signature. That is to say, AE schemes achieve the security requirements of confidentiality, integrity, authenticity, and non-repudiation. So far, a number of AE schemes and variations have been proposed [3]-[16].

In AE schemes, only a designated verifier is able to recover the signed message and then verify the validity of corresponding signatures. In case of a later dispute over repudiation, it is necessary to either reveal the verifier's private key or perform a zero-knowledge protocol for convincing any third party of the signer's dishonesty. In 1998, Araki *et al.* [5] proposed a convertible limited verifier signature scheme without revealing the designated verifier's private key or performing the zero-knowledge protocol. Their scheme can be recognized as a new type of AE scheme and called the convertible authenticated encryption (CAE) scheme.

In 2008, Wu *et al.*, [9] elaborated on the merits of the CAE and the multisignature schemes to propose a convertible

multi-authenticated encryption (CMAE) scheme for group-oriented applications based on the computational Diffie-Hellman problem (CDHP). Their scheme allows a group of signers to cooperatively produce a valid authenticated encryption signature and still preserves the characteristic of CAE schemes. The signed message is recovered from a given signature and the validity of the recovered message along with its corresponding signature can be verified by checking the message redundancy embedded in the message. That is, the security of Wu *et al.*'s scheme is primarily based on the adopted message redundancy generation algorithm. Recently, Tsai [10] proposed a new CMAE scheme with one-way hash function. His scheme outperforms Wu *et al.*, one in terms of computational efficiency.

The security of Tsai's scheme is based on the one-way hash function and solving the discrete logarithm problem, which are believed infeasible to solve in polynomial time. However, we will show that the scheme cannot provide the semantic security of the message. In this paper, we gave a further improvement to provide the semantic security. Our improvement makes it more efficient in computation and communication.

## II. REVIEW OF TSAI'S SCHEME

The Tsai's scheme manipulated over  $GF(p)$ , is comprised of three phases: 1) Signature encryption phase, 2) Message recovery and verification phase, and () Signature conversion phases. Before reviewing of the Tsai's scheme, all necessary parameters are described as follows:

$p, q$ : large primes, such that  $q|(p-1)$ ,  
 $g$ : a generator of order  $q$  over  $GF(p)$ ,  
 $h()$ : a public one way hash function,  
 $\oplus$ : the bit-wise exclusive-or operation,  
 $U_i$ : denote a user.

Each  $U_i$  owns the private key  $x_i \in_R Z_q^*$  and public key  $y_i = g^{x_i} \bmod p$  which is publicly accessible. Each phase of Tsai's scheme is described as follows.

### A. Signature Encryption (SE) Phase

Without loss of generality, let  $SG = \{U_1, U_2, \dots, U_n\}$  be the signing group and signers  $U_i \in SG$  want to send  $U_v$  a message  $M$ , where  $1 \leq M \leq p-1$ . For signing the message  $M$ , each  $U_i \in SG$  performs the following steps:

Step 1:  $U_i$  chooses a random number  $w_i \in Z_q^*$  to compute and broadcasts  $r_i$  to  $U_j \in SG \setminus \{U_i\}$ .

$$r_i = g^{w_i} \bmod p \quad (1)$$

Step 2: By all received  $r_j$ 's from  $U_j \in SG \setminus \{U_i\}$  and  $r_i$ ,  $U_i$

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computes and then sends  $s_i$  to the  $U_j \in SG \setminus \{U_i\}$

$$R = M \left( \prod_{U_j \in SG} r_j \right) \bmod p, \quad (2)$$

$$K = h(R, M) \bmod p, \quad (3)$$

$$s_i = x_i K + w_i \bmod q \quad (4)$$

Step 3: Upon receiving  $s_j$  from  $U_j \in SG \setminus \{U_i\}$ ,  $U_i$  verifies

$$g^{s_j} = y_j^K r_j \bmod p. \quad (5)$$

If Eq. (4) holds, proceed to Step 4; otherwise  $U_i$  requests  $U_j \in SG \setminus \{U_i\}$  to resend  $s_j$ .

Step 4: When all  $\{r_j, s_j\}$ 's are collected and verified, the clerk  $U_k$  who can be any signer in  $SG$ , chooses  $d \in_R Z_q$  to compute

$$S = \sum_{U_j \in SG} s_j \bmod q, \quad (6)$$

$$C_1 = g^d \bmod p, \quad (7)$$

$$C_2 = R \oplus (y_v^d \bmod p), \quad (8)$$

Note that  $y_v$  is the public key of the designated recipient  $U_v$ .

Step 5: The clerk  $U_k$  sends  $\{S, C_1, C_2, K\}$  to the designated recipient  $U_v$ .

Tsai's scheme is a distributed scheme and, hence, anyone in  $SG$  can act as the clerk to collect and verify all individual signatures, and combines them into  $\{S, C_1, C_2, K\}$  for the designated recipient  $U_v$ . It means that each individual signers can produce the same signature  $\{S, C_1, C_2, K\}$ . If the clerk dishonestly performs his tasks, the validity of the combined  $\{S, C_1, C_2, K\}$  will be detected by other signers.

#### B. Message Recovery and Verification (MRV) Phase

Upon receiving  $\{S, C_1, C_2, K\}$  from the clerk  $U_k$ , the designated recipient  $U_v$  can perform as following three steps: Step 1: The designated recipient  $U_v$  computes

$$R = C_2 \oplus C_1^{x_v} \bmod p. \quad (9)$$

Step 2: The recipient  $U_v$  recovers the message  $M$  by computing

$$M = R (g^{-s}) \left( \prod_{U_i \in SG} y_i \right)^K \bmod p. \quad (10)$$

Step 3: The recipient  $U_v$  verifies

$$K = h(R, M). \quad (11)$$

$$K = h \left( M \left( g^{-s} \left( \prod_{U_i \in SG} y_i \right)^K \right)^{-1} \bmod p, M \right). \quad (12)$$

If Eq. (11) holds, the signature is accepted; otherwise it is refused.

#### C. Signature Conversion (SC) Phase

If dispute on repudiation, the recipient  $U_v$  can release the  $\{S, K\}$  for the message  $M$ . With this converted signature, anyone can validate its validity by computing

#### D. Weakness of Tsai's Scheme

Suppose an adversary gets a valid  $\{K, S, y_i; i = 1, 2, \dots, n\}$ , he can check whether his guessed message  $M^*$  satisfies Eq.

(12). If it holds, then he gets the actual message. So Tsai's scheme cannot provide the semantic security of the message. Tsai's scheme cannot provide the semantic security of the message.

### III. OUR IMPROVED SCHEME

Semantic security is of very importance to an authenticated encryption scheme for practical communications. Otherwise, if the possible messages are limited, then an adversary can eventually determine which message the signer signs by checking which satisfies the verification equalities. Tsai's scheme cannot provide the semantic security of the message. To provide the semantic security, the authors propose an improved scheme. Furthermore, our improvement is more efficient in term of computational efforts. Each phases of our improved scheme are described as follows.

#### A. Signature Encryption (SE) Phase

The system model and parameters are defined as those in Section II. For signing the message  $M$ , each  $U_i \in SG$  performs the following steps:

Step 1:  $U_i$  chooses a random number  $w_i \in Z_q^*$  to compute

$$r_i = g^{w_i} \bmod p \quad (13)$$

And broadcasts  $r_i$  to  $U_j \in SG \setminus \{U_i\}$ .

Step 2: By all received  $r_j$ 's from  $U_j \in SG \setminus \{U_i\}$  and  $r_i$ ,  $U_i$  computes

$$R = M \oplus \left( \prod_{U_j \in SG} r_j \right) \bmod p, \quad (14)$$

$$K = h \left( \prod_{U_j \in SG} r_j, M \right) \bmod p, \quad (15)$$

$$s_i = x_i K + w_i \bmod q, \quad (16)$$

And then sends  $s_i$  to the  $U_j \in SG \setminus \{U_i\}$ .

Step 3: Upon receiving  $s_j$  from  $U_j \in SG \setminus \{U_i\}$ ,  $U_i$  verifies

$$g^{s_j} = y_j^K r_j \bmod p. \quad (17)$$

If Eq. (4) holds, proceed to Step 4; otherwise  $U_i$  requests  $U_j \in SG \setminus \{U_i\}$  to resend  $s_j$ .

Step 4: When all  $\{r_j, s_j\}$ 's are collected and verified, the clerk  $U_k$  who can be any signer in  $SG$ , chooses  $d \in_R Z_q$  to compute

$$S = \sum_{U_j \in SG} s_j \bmod q, \quad (18)$$

$$C_1 = g^d \bmod p, \quad (19)$$

$$C_2 = R \oplus (y_v^d \bmod p), \quad (20)$$

$$Q = K \oplus (y_v^d \bmod p). \quad (21)$$

Note that  $y_v$  is the public key of the designated recipient  $U_v$ .

Step 5: The clerk  $U_k$  sends  $\{S, C_1, C_2, Q\}$  to the recipient  $U_v$ .

#### B. Message Recovery and Verification (MRV) Phase

Upon receiving  $\{S, C_1, C_2, Q\}$  from the clerk  $U_k$ , the designated recipient  $U_v$  can perform as following three steps:

Step 1: The designated recipient  $U_v$  computes

$$R = C_2 \oplus C_1^{x_v} \text{ mod } p, \quad (22)$$

$$K = Q \oplus C_1^{x_v} \text{ mod } p. \quad (23)$$

Step 2: The recipient  $U_v$  recovers the message  $M$  by computing

$$M = R \oplus (g^S (\prod_{U_j \in SG} y_j)^{-K} \text{ mod } p). \quad (24)$$

Step 3: The recipient  $U_v$  verifies

$$K = h((g^S (\prod_{U_j \in SG} y_j)^{-K} \text{ mod } p), M). \quad (25)$$

If Eq. (25) holds, the signature is accepted; otherwise it is refused.

### C. Signature Conversion (SC) Phase

If dispute on repudiation, the recipient  $U_v$  can release the  $\{S, K\}$  for the message  $M$ . With this converted signature, anyone can validate its validity by Eq. (25).

## IV. ANALYSIS OF THE IMPROVED SCHEME

Security analysis and computational complexity analysis of our improved scheme are given below.

### A. Security Analysis

The security of the proposed scheme is based on well-known cryptographic assumptions: solving the discrete logarithm problem (DLP) [17] and the intractability of reversing the one-way hash function (OWHF) [18]. In the following, we discuss some possible attacks against the proposed scheme and show that the proposed scheme is secure under the protection of the DLP and OWHF assumptions.

1) *Can the attacker reveal the secret key  $x_i$  of the signer  $U_i \in SG$  or the secret key  $x_v$  of the designated recipient  $U_v$  from all public information?*

With the authenticated encryption signature  $\{S, C_1, C_2, I, Q\}$ ,  $s_i$  and  $r_i$ , the attacker cannot derive the signer's secret key  $x_i$  from Eq. (16), since the equation contains two unknown variables  $x_i$  and  $w_i$ , and  $w_i$  is protected under the DLP assumptions. Similarly, the attacker cannot reveal verifier's secret key  $x_v$  from Eq.(22) and Eq.(23), since  $x_v$  here is protected under the DLP assumptions.

2) *Can the attacker forge an authenticated encryption signature?*

To forge a signature for satisfying Eq. (24), the attacker must know the all random numbers  $w_i$ 's and  $U_i$ 's private key  $x_i$ . However, it will not be workable since all  $w_i$ 's and  $x_i$  are protected under the DLP assumption. The attacker cannot get them, because  $w_i$  and  $x_i$  are only hold by the signer  $U_i \in SG$ . Thus, it is impossible for any attacker to forge the digital multi-signature of the message  $M$ .

3) *Can the attacker forge a converted signature?*

If an adversary wants to forge a signature  $\{S, K\}$  of the message  $M$ , this adversary must know the random number  $w_i$ ,

$U_i$ 's private key  $x_i$ , the message  $M$  and  $R$ . Assume that this adversary is an outsider. He cannot get them, because the  $w_i$  and  $x_i$  are only hold by the signer  $U_i$ , and  $R$  is the authenticated message for the message  $M$ . Assume that this adversary is an insider. He cannot get the  $w_i$  and  $x_i$ , because the  $w_i$  and  $x_i$  are only hold by  $U_i$ . Thus, it is impossible for any adversary to forge the digital multi-signature of the message  $M$ .

4) *Can the attacker recover the message from the authenticated encryption signature?*

With the authenticated encryption signature  $\{S, C_1, C_2, I, Q\}$ , the attacker cannot derive the message  $M$  from Eq. (25) since the  $K$  is protected under the DLP assumption. In addition, the  $x_v$  is only hold by the signer  $U_v$ . The attacker cannot derive the  $R$  from Eq. (22). Thus, it is impossible for an attacker to recover the message  $M$  from the Eq. (24) successfully.

TABLE I: COMPARISONS OF COMPUTATIONAL COMPLEXITIES

Phases	Our proposed scheme	Tsai's scheme [2]
SE	Time complexities	$nT_H + (3n)T_{EXP} + (n^2+n-1)T_{MUL}$
	Rough Estimation	$(n^2+725n-1)T_{MUL}$
MRV	Time complexities	$T_H + (n+1)T_{MUL} + 3T_{EXP} + T_{INV}$
	Rough Estimation	$(n+734)T_{MUL}$
SC	Time complexities	$T_H + nT_{MUL} + 2T_{EXP} + T_{INV}$
	Rough Estimation	$(n+494)T_{MUL}$
Total	Time complexities	$(n+2)T_H + 2T_{INV} + (n^2+3n-1)T_{MUL} + (3n+5)T_{EXP}$
	Rough Estimation	$(n^2+727n+1227)T_{MUL}$

5) *Can the attacker verify the signature before converted?*

It requires the message  $M$  to perform the signature verification of Eq. (25). From the discussion (4), the attacker cannot obtain the message  $M$  before the signature is converted. Hence he cannot verify the signature.

### B. Computational Complexity Analysis

We denote the following notations to facilitate the performance evaluation:

$T_{MUL}$ : time for performing a modular multiplication computation,

$T_{EXP}$ : time for performing a modular exponentiation computation,

$T_H$ : time for performing a one-way hash function computation,

$T_{INV}$ : time for performing a modular inversion computation.

The time for performing the modular addition and the exclusive OR (XOR) operation are ignored, since they are relatively smaller than those for performing other operations. From [19], [20], the time complexities can be respectively regarded as  $T_{EXP} \approx 240 T_{MUL}$ ,  $T_{INV} \approx 10 T_{MUL}$ , and  $T_H \approx 4 T_{MUL}$ . The performance evaluations of the two schemes are

described as Table I. Thus, we conclude that the proposed scheme is more efficient than Tsai's in term of computational complexity.

## V. CONCLUSION

Semantic security is of very importance to an authenticated encryption scheme for practical communications. This article has shown that Tsai's scheme cannot provide the semantic security of the message. To provide the semantic security, the authors propose a secure CMAE scheme based on the intractability of one-way hash functions and discrete logarithms. Furthermore, our improvement is more efficient in term of computational complexity.

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