Cover Time Analysis of Ad-Hoc Wireless Network

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***Abstract*—In this work our focus is on the cover time analysis of an ad-hoc wireless network, when the nodes are allowed to move with in an compact space. We consider an ad-hoc network in 2-dimensional space, and a sequence $X_n = \{X_1, X_2 \ldots, X_n\}$, with *n* nodes distributed uniformly over a compact space $C \subset R^2$. We derive a strong law result for the cover time of the network and show that for transmission radius $r_n = \sqrt{\frac{\log n}{\pi n}}$ he critical cover time C_n is at most of order of $o(n^{5/2})$.**

***Index Terms*—Ad-hoc networks, capacity, mobility, cover-time.**

I. Introduction

In this paper, we study the cover time in mobile ad hoc network. The line of study starts with the seminal work of Gupta and Kumar [1], where they proved the throughput capacity of wireless ad-hoc network follows $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$, when the nodes are uniformly distributed. In Gupta and Kumar [7], derived for critical transmission range $\sqrt{\frac{\log n}{\pi n}}$ the network is asymptotically connected with probability approaching to one. After Gupta and Kumar, a significant contribution in this direction was made by Grossglauser and Tse, in [4], authors examined the asymptotic throughput capacity of large mobile ad-hoc networks. They showed that direct communication between sources and destinations alone cannot achieve high through put. Grossglauser and Tse [4] propose to spread the traffic to intermediate relay nodes to exploit the multiuser diversity benefits of having additional "routes" between a source and a destination. Grossglauser and Tse [4] considered a mobile ad-hoc network and proved that the traffic carrying capacity of an ad-hoc network significantly increase when the network have mobile nodes. Grossglauser and Tse, also showed that the capacity is independent of node density.

Latter on, Xiaojun Lin, Gaurav and others,[6] studied the fundamental tradeoff between delay and capacity for a mobile ad-hoc network under the Brownian motion model. In [5], Gaurav, Mazumdar and others studied the delay-capacity trade-offs in mobile ad-hoc networks. They introduced the notion of critical delay and showed that the critical delay is inversely propor- tional to the characteristic path length. Gamal, Mammen and others, in [3] gives a scheme that achieves the optimal throughput-delay trade-off by varying number of hops, transmission range and degree of node mobility.

Yu and Kim [2], studied the same model suggested by Gupta and Kumar in [1], with additional condition of node mobility. Yu and Kim [2], relate mean delay with partial cover time. They derive that the mean delay is per source-destination pair as $\Theta(n)$ or x and the worst case delay is $\Theta(n\log n)$ or $\Theta\left(\frac{\sqrt{n}\log n}{v}\right)$, corresponding to one slot time length that is either constant or $\frac{1}{v\sqrt{n}}$, where *n* is the number of nodes in the network and *v* is constant speed of the nodes.

Yu and Kim in [2] have an assumption of homogenous network. For being a homogenous $E[Y_i]$ must satisfy following conditions: For any given $\in$

$$E[Y_i] = \frac{n-(i-1)}{n} \text{ and } P\left[\left|Y_i - \frac{n-(i-1)}{n}\right| \geq \in\right] = 0$$

if a network doesn't satisfies above conditions, is known as non-homogeneous network. Yu and Kim derive that the mobile cover time of the network in both case is order of $n\log(n)$.

In our model we are relaxing the above mentioned assumption.

Here, we adopt the same protocol model as in Gupta and Kumar [1] and Yu and Kim [2]. Results in this paper are in continuation of the Yu and Kim [2].

II. Our Model

We consider an ad-hoc network in 2-dimensional space, with *n* nodes distributed uniformly over a compact space $C \subset R^2$. Without loss of generality we can assume compact space $C \subset R^2$, as a disk of unit area. Also these n nodes are allowed to move inside the unite disk with a constant speed, v meters per second. We assume that each node in the network can be source node for one session and destination node for another session. We also assume source - destination association does not change with time, although nodes themselves can change their location with time on the unit disk. A packet can transmit directly from the source node to its destination node or they can go through one or more other nodes servicing as relay node,

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depending upon the transmission range r_n of the source node. We assume constant transmission range r_n for each node in the network and consider as a function of number of nodes in the network. A node be covered by a packet from a source node if the node receives the packet. Let a node x having a packet (started from source node) forward it to a next node in his transmission range r_n. In case there are more then one node in the transmission range of node x, the next node for forwarding the packet will be selected randomly from nodes in the transmission range of node x. This means that the covered node may receive the packet and forward it to any other node in its neighborhood.

III. COVER TIME.

The cover time define as the number of transmission until all nodes in the network became covered by a given packet started from the source node.

Let x_i be the number of transmission required to cover the *i*th node, given *i*-1 nodes of the network are already covered by the packet from a source node. Also let Y_i be the event of successful coverage of *i*th node from $i-1$th covered node in single transmission attempt.

For sufficiently large network, the number of nodes lying in r_n- neighborhood of a point follows the Poisson distribution with parameter πr_n^2, where r_n be the transmission range of a node in the network. Let p_i is the probability Y_i .Then

$$\begin{aligned} p_i &= P\ Y_i \\ &= \text{P[coverage of ith node from } i-1\text{th node in single attempt]} \\ &= \text{P[at least one uncovered node in } r_n \text{ nbd of } i-1\text{th node.]} \\ &= 1-e^{-\pi r_n^2} \end{aligned} \quad (1)$$

Since p_i is independent of i we have *Yi's* are independent and identically distributed. (Note: From here on ward we use p instead of p_i.

If the critical transmission range r_n is sufficiently small and converges to 0, for sufficiently large n. Then

$$p \sim \pi r_n^2 \quad (2)$$

Clearly for fixed *p*, we have X_i be the a geometric random variable with parameter *p, i.e.,*

$$P[X_i = x] = p(1-p)^{x-1}, \qquad x = 1,2,3,\ \ldots,\infty$$

where *p* be the probability of event Y_i. Let C_n be the mobile cover-time of a network having *n* nodes.

$$\begin{aligned} E[C_n] &= E[X_1 + X_2 + \ldots + X_n] \\ &= E\,[E[X_1 + X_2 + \ldots + X_n \mid Y_1, Y_2, \ldots, Y_n]] \\ &= E\,[E[X_1 \mid Y_1] + E[X_2 \mid Y_2] + \ldots + E[X_n \mid Y_n]] = \sum_{i=1}^{n} E\,[E[X_i \mid Y_i]] \end{aligned}$$

Since $X_i \mid Y_i, \forall_i = 1, 2, \ldots, n$ are independently distributed. Than

$$E[C_n] = \sum_{i=1}^{n} E\left[\frac{1}{P}\right] = \frac{n}{p} \quad (3)$$

Using (2) in (3), we get

$$E[C_n] = \frac{n}{\pi r_n^2} \quad (4)$$

Similarly,

$$\begin{aligned} E[C_n^2] &= E[(X_1 + X_2 + \ldots + X_n)^2] \\ &= \mathrm{E}\left[(\sum_{i=1}^{n} x_i)^2\right] = n^2 E\left[X_1^2\right] \end{aligned} \quad (4)$$

Since *Xi's* are mutually exclusive events, i.e., $X_i \cap X_j = \phi\ \forall_i \neq j$.

$$\begin{aligned} E[C_n^2] &= n^2 E\left[E\left[X_i^2 \middle| Y_1, Y_2, \ldots, Y_i\right]\right], \quad i = 1,2,\ldots,n \\ &= n^2 E\left[E\left[X_i^2 \middle| Y_i\right]\right] \end{aligned}$$

Since X_i is independent of $Y_j, \forall_j = 1, 2, \ldots, i-1$ are independently distributed. Than

$$E[C_n^2] = n^2 E\left[\frac{2-p}{p^2}\right] = \frac{n^2(2-p)}{p^2} \quad (5)$$

Using (2) in (5), we get

$$E[C_n^2] = \frac{n^2(2-\pi r_n^2)}{\pi^2 r_n^4} < \frac{2n^2}{\pi^2 r_n^4} \quad (6)$$

$$V\,[C_n] = \frac{n^2(2-\pi r_n^2)}{\pi^2 r_n^4} - \left[\frac{n}{\pi r_n^2}\right]^2 \quad (7)$$

The following result gives the critical cover time above which the network is complete covered almost sure. Here we consider the same critical transmission range $r_n = \sqrt{\frac{\log\ n}{\pi n}}$ as suggested in Gupta and Kumar [7].

Theorem 3.1 Let transmission range $r_n = \sqrt{\frac{\log\ n}{\pi n}}$, then the mobile cover time Cn of the random network is atmost $cn^{5/2+\delta}$, almost surely, i.e.,

$c_n \le cn^{5/2+\delta}$, almost surely,

where *c*, and δ is an arbitrary small positive real number.

Proof. By Markovs inequality, we have

$$|p\ c_n > k \le \frac{E\left[C_n^2\right]}{k^2} \quad (8)$$

Using (6) in (8), we get

$$p\ c_n > k\ \leq \frac{2n^2}{\pi^2 k^2 r_n^4} \tag{9}$$

Substitute, $k = cn^{5/2+\delta}, c, \delta > 0$ and transmission radius $r_n = \sqrt{\frac{\log\ n}{\pi n}}$ in the above expression, we get

$$p\left[c_n > cn^{5/2+\delta}\right] \leq \frac{1}{n^{1+2\delta}(\log n)^2} \tag{10}$$

The above probability is summable, i.e.,

$$\sum_{n=1}^{\infty} p\left[c_n > cn^{5/2+\delta}\right] < \infty$$

Then by Borel-Cantelli's Zero-One Law, we have

$$p\left[c_n \leq cn^{5/2+\delta}, \quad i.o\right] = 1. \tag{11}$$

This implies that $c_n \leq cn^{5/2+\delta}$ (12)
almost surely,
where c, and δ is an arbitrary small positive real number.

From the above theorem it is clear that the cover time Cn of a mobile network can't be more that of order of $n^{5/2}$.

IV. Conclusion

We are giving an analytic result for a network with stationary nodes and we do not have any topological information about the node. Our main result is an strong law result, gives an almost sure upper bound of the cover time of given wireless network. We showed that even in worst case the cover time of the network is of order of $o(n^{5/2})$, where n be the number of the nodes in the network.

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