Median Filtering Frameworks for Reducing Impulse Noise from Grayscale Digital Images: A Literature Survey

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Abstract—One of the noise types that is normally degrades digital images, including grayscale digital images, is impulse noise. Therefore, researches regarding to impulse noise removal have become one of the active researches in the field of image restoration. Median based filter is normally becoming the choice to deal with this type of noise. However, there are many variations of median filter in literature. In addition to standard median filter, there are weighted median filter, iterative median filter, recursive median filter, directional median filter, switching median filter, and adaptive median filter. Therefore, this paper will survey these median filtering frameworks.

Index Terms—Impulse noise, median filter, standard median filter, weighted median filter, iterative median filter, recursive median filter, directional median filter, switching median filter, adaptive median filter.

I. INTRODUCTION

Similar to other digital signal, digital images are sometime could be corrupted by noise. One of the noise types normally related to digital image is impulse noise. Impulse noise can be mathematically modelled by several equations [1]. In general, impulse noise appears as a sprinkle of bright or dark spots on the image, and normally these spots have relatively high contrast towards their surrounding areas. Therefore, even at low corruption level, impulse noise can significantly degrade the appearance and quality of the image [2], [3].

A popular solution to deal with impulse noise is by using rank-order filters, or also known as order-statistic filters. This type of filters is nonlinear and works in spatial domain. It uses sliding window approach, where on each sliding-iteration, only the value of the pixel corresponds to the centre of the window is changed. This value is obtained based on the ordered intensity values of the pixels contained in the area defined by the filtering window [4], [5].

Among these rank-order filters, median based filters are the most popular techniques to reduce both bipolar and uni-polar impulse noise [4], [5]. Generally, median filter uses median value in its filtering process. The median value \( \bar{X} \) of a sample is defined as [6]:

\[
\bar{X} = \begin{cases} 
X_{(n_i+1)/2} & : \ n_i \text{ is odd} \\
0.5(X_{n_i/2} + X_{(n_i+1)/2}) & : \ n_i \text{ is even}
\end{cases}
\]

where \( X_1, X_2, ..., X_n \) are the intensity values, arranged in either increasing or decreasing order, and \( n \) is the size of the sample. However, there are a lot of median filter variations. Therefore, this paper reviews some of the median filter frameworks.

II. FRAMEWORKS OF MEDIAN FILTERS

Currently, there are thousands median based filters available in literature. For example, the search “median filter” from Google and IEEEExplore database returns thousands of results. It is impossible to describe each method in details. Therefore this paper surveys some common frameworks used by these filters. It is worth noting that some of the methods are using more than one framework stated in this paper.

A. Standard Median Filter (SMF)

Standard median filter (SMF), or also known as median smoother, has been introduced by Tukey in 1971 [7]. The filtered image \( F = \{F(i,j)\} \) from SMF can be defined by the following equation [5]:

\[
F(i,j) = \text{median}_{(k,l)\in W_{h\times w}} \{D(i+k,j+l)\}
\]

where \( W_{h\times w} \) is a sliding window of size \( h\times w \) pixels centred at coordinates \((i, j)\). The median value is calculated by using equation (1) with \( n = h\times w \).

Although SMF can significantly reduce the level of corruption by impulse noise, uncorrupted pixel intensity values are also altered by SMF. This undesired situation happens because SMF does not differentiate between uncorrupted from corrupted pixels. Furthermore, SMF requires a large filter size if the corruption level is high. Yet, large filter of SMF will introduce a significant distortion into the image [8].

It is worth noting that equation (1) is normally using sorting algorithm such as quick-sort or bubble-sort to arrange the samples in increasing or decreasing order. Even though sorting algorithm can be easily implemented, sorting procedure requires long computational time when \( W_{h\times w} \) is a large filter because the number of samples (i.e. \( n = h\times w \)) is big. Thus, in order to avoid from using any direct sorting algorithm, the use of local histograms has been proposed for median value calculation. The time required to form local histogram can be reduced by using a method proposed by Huang et al. [9], where instead of updating \( h\times w \) samples, only \( 2h \) samples need to be updated in each sliding-iteration.
B. Weighted Median Filter (WMF)

One of the branches of median filter is weighted median filter (WMF). WMF was first introduced by Justusson in 1981, and further elaborated by Brownrigg. The operations involved in WMF are similar to SMF, except that WMF has weight associated with each of its filter element. These weights correspond to the number of sample duplications for the calculation of median value. The filtered image \( F = \{ F(i,j) \} \) from WMF can be defined by the following equation [8]:

\[
F(i,j) = \text{median}_{(k,l)\in W_{h,w}} \left[ W_{h,w}(k,l) \otimes D(i+k, j+l) \right]
\]

where operator \( \otimes \) indicates repetition operation. The median value is calculated using equation (1) with \( n \) is equal to the total of \( W_{h,w}(k,l) \). Normally, the filter weight \( W_{h,w} \) is set such that it will decrease when it is located away from the centre of the filtering window. By doing so, it is expected that the filter will give more emphasis to the central pixel, and thus improve the noise suppression ability while maintaining image details [10-13]. However, the successfulness of weighted median filter in preserving image details is highly dependent on the weighting coefficients, and the nature of the input image itself. Unfortunately, in practical situations, it is difficult to find the suitable weighting coefficients for this filter, and this filter requires high computational time when the weights are large [14-16].

Some researchers, such as [8], [17], proposed adaptive weighted median filters (AWMF), which is an extension to WMF. By using a fixed filter size \( W_{h,w} \), the weights of the filter will be adapted accordingly base on the local noise content. This adaptation can be done in many ways, mostly based on the local statistics of the damaged image. For example, in [17], the weights of the filter are defined as:

\[
W_{h,w}(j,k) = \left\{ \begin{array}{ll}
w_{h,w}(0,0) - c \left( \frac{\sigma^2 d}{\bar{x}} \right) & \text{if } (j,k) \text{ is the central filter element} \\
0 & \text{otherwise}
\end{array} \right.
\]

where \( W_{h,w}(0,0) \) is a preset weight for the central filter element, \( c \) is a preset scaling factor, \( d \) is the distance of location \((j,k)\) to coordinates \((0,0)\), and \( \sigma^2 \) and \( \bar{x} \) are the local variance and local mean, respectively, defined by a sliding window of size \( h \times w \) pixels. The operator \( < \rightarrow > \) presents the rounding operation if the argument inside it is a positive value. Otherwise it will truncate the value to zero.

Centre weighted median filter (CWMF) is a special type of WMF. CWMF has the weights defined as follow:

\[
W_{h,w}(k,l) = \left\{ \begin{array}{ll}
n_{w} & \text{if } (k,l) = (0,0) \\
1 & \text{otherwise}
\end{array} \right.
\]

where \( n_w \) is an odd integer, with value greater or equal to one. Coordinates \((k,l) = (0,0)\) presents the centre of the filter. When \( n_w \) is set to one, CWMF becomes SMF. Large value of \( n_w \) is good in preserving details but worse in noise cancellation. When \( n_w \) is greater or equal to \( h \times w \) (i.e. the area covered by filter \( W_{h,w} \)), CWMF turns into the identity filter. In this condition, CWMF does not filter the image, and thus the output image will become exactly the same as its corresponding input [12].

C. Iterative Median Filter

In several impulse noise filtering methods, such as [18-21], require iterative filtering procedure in their implementation. Iterative method requires the same procedure to be repeated several times. In general, iterative median filter with \( n \) iterations, requires \( n - 1 \) temporary images. Iteration procedure enables median filtering process to use smaller filter size and reduce the computational time, while maintaining local features or edges of the image. The number of iterations \( n \) can be set by the user, or the iteration process stops when the output image converged (i.e. the current output image is equal to the previous output image). In practical, the number of iterations needed is dependent to the level of corruption and also the nature of the input image itself.

D. Recursive Median Filter

Several researches in median filtering, such as [16], [74-78], use recursive approach in their methodology. Theoretically, recursive median filters can be considered analogous to infinite impulse response (IIR) filter because their outputs at certain position are determined only not from the input intensities, but also from the calculated outputs at previous locations. In implementation of recursive median filter, normally the degraded image and the filtered image share the same data array.

In this method, the already processed pixels are now considered as noise free input pixels. Thus, by replacing the input pixels with these values, it assumes that the median value calculation will be more accurate. However, if the filter fails to remove the noise at previous locations, the error might be propagated to other area of the image. Furthermore, it is worth noting that the result from recursive median filter is dependent to the direction of filtering.

E. Directional Median Filter

Directional median filter, or also known as stick median filter, works by separating its 2-D filter into several 1-D filter components [22-24]. Each filter component or stick, presented as a straight line, corresponds to a certain direction or angle \( \theta \). For a window of size \( h \times w \) pixels, there are \( h+w-2 \) sticks that will be used. The computed median values from these 1-D filters are then combined to obtain the final result. In [24], the output intensity is defined as:

\[
F(i,j) = \max_{(k,l) \in W_{\theta}} \left\{ \text{median}_{(k,l) \in W_{\theta}} \left[ D(i+k, j+l) \right] \right\}
\]

where \( \theta \) is the stick. Here, the output intensity is defined as the largest median value determined at each location.

F. Switching Median Filter

Nowadays, one of the popular median filtering approaches is switching median filter, or also known as decision based median filter. This approach has been used in recent works, such as [25-30]. Switching median filter tries to minimize the undesired alteration of uncorrupted pixels by the filter. Therefore, in order to overcome this problem, switching median filter checks each input pixel whether it has been
corrupted by impulse noise or not. Then it changes only the intensity of noisy pixel candidates, while left the other pixels unchanged. Normally, switching median filter is built from two stages. The first stage is for noise detection, while the second stage is for noise cancellation.

The output from the noise detection stage is a noise mask $M$. This mask is a binary mask, and normally defined as follow:

$$M(i, j) = \begin{cases} 1 & \text{impulse noise candidate} \\ 0 & \text{otherwise} \end{cases}$$

Noise detection procedure used by researchers are normally depending on the noise model been used. For fixed-valued impulse noise (i.e. salt-and-pepper noise), mostly the noise detection is done by thresholding the intensity values of the damaged image. Other popular noise detection methods include by checking the difference between intensity of the current pixel with its surrounding, inspecting the difference of the damaged image with its median filtered versions, or by applying some special filters. Next, mask $M$ will be used in the noise cancellation stage, where only pixels with $M = 1$ are processed by the median filter. For the calculation of median, only "noise-free" pixels (i.e. pixels with $M = 0$) are taken as the sample.

G. Adaptive Median Filter

Actually, the concentration of impulse noise on an image is varied because impulse noise is a random noise. Therefore, there are regions of the image with high level of corruption, and there are also regions with low level of corruption. For an effective noise filtering process, a larger filter should be applied to regions with high level of corruption. In contrast, a smaller filter should be applied to regions with low level of corruption. Therefore, many works, such as [25], [26], [28-32], have proposed methods that are able to adjust the size of the filter accordingly based on the local noise content. Because the size of the filter is adapted to the local noise content, this type of median filter is known as adaptive median filter.

Commonly, the filter size at each processing locations is initially set to 3x3. The size of the filter is then gradually expanding until it met certain criteria. These criteria can include the number of potential noise free pixels, local mean, local maximum value, local minimum value or local median value. Sometimes, these criteria can never be met. Therefore, some methods restrict the expansion of the filter up to certain size only. Although adaptive median filters are good in restoring image corrupted by impulse noise, these filters normally require considerably long computational time when the image is highly corrupted.

H. Median Filter Incorporating Fuzzy Logic

In order to preserve the local details of the image, median filter should only change the intensity of corrupted pixels on the damaged image. However, it is very difficult to detect the corrupted pixels from this image correctly. Even for fixed-valued impulse noise (i.e. salt-and-pepper noise), where the noise only takes values 0 and 1, simple thresholding method still cannot classify the pixels effectively. This is because some of the uncorrupted pixels are also been presented by these two values. Thus, researches such as [14], [23], and [33-36], incorporate fuzzy logic approach into median filtering process.

There are several ways on how fuzzy logic been used in median filtering process. Fuzzy logic can be used to grade how high a pixel has been corrupted by impulse noise. Normally, based on this fuzzy degradation measure, a proper correction will be applied. On the other hand, some of the methods use fuzzy logic as a decision maker that selects a proper filter, from a filter bank, for a given input image.

In order to use fuzzy logic, the damaged image must first undergo a fuzzification process. Normally, the input for the fuzzification process is the intensity of the pixels, or the intensity differences between the current pixels with its surrounding. The system then executes the noise filtering process based on the fuzziness values obtained. The results are then found through a defuzzification process.

Although fuzzy logic can improve impulse noise suppression, methods such as [33-35] use too many fuzzy rules to obtain an acceptable result. As a consequence, this condition makes their filtering methods becoming computational expensive. Furthermore, their restoration results are also too dependent to the number of membership functions, and also to the parameters that control the shape of the membership functions. Therefore, such methods are difficult to be implemented as an automatic impulse noise reduction filter, and also cannot be used for real-time processing.

III. SUMMARY

This paper surveys eight common median filtering frameworks. Each framework has its own advantages, and disadvantages. From literature, we found that most of the recent median filtering based methods employ more than two of these frameworks in order to obtain an improved impulse noise cancellation.

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REFERENCES


