Crystal Fiber Based Erbium Doped Amplifiers and Their Gain

Peiying Chen, Suqin Feng , Lanzhu An , Haixu Song , Ximei Zhao, and Cunge Xu

***Abstract*—The paper is based on the numerical solution of Euler's method to deal with the initial value problems for Erbium doped fiber amplifiers'（EDFAs）rate equations, and their gain dependences are discussed. Simulation of the propagation of the signal and pump power versus the amplifier length has been done. Modifying the input pump power and input signal power has obtained the gain profile of the PCF based EDFAs. Fiber designs which allow us to have large gain are presented and demonstrated that the PCF based Erbium doped amplifiers may deliver gain 54dB with small losses. The gain is greater than the conventional EDFAs.**

***Index Terms*—Photonic crystal fiber, erbium doped fiber amplifiers, gain.**

I. Introduction

The term "amplifiers" refer to boost its transmission of information along the fiber. Over a certain distance, information cannot be transmitted through the fiber without optical amplifiers. Fiber attenuation limits the strength of optical signal. Power amplifiers are employed to enlarge the power of the signal before it approaches the fiber transmission link and it can extend the transmission distance. Power amplifier has high gain while preamplifier needs lower noise value and the line amplifier requires both of them. Erbium—doped fiber amplifiers (EDFAs) can be made as optical amplifiers. After the first paper on EDFAs published in 1987, optical communications has begun its new era. EDFAs become common and popular now, because Erbium atoms provide light amplification over the range of wavelength 1525—1610 nm.

The objective of this paper is to develop a practical method to perform the gain for Erbium Doped Fiber Amplifiers based on Photonic Crystal Fibers on the pump and signal power in co-propagating pump configuration. Simulation is based on the numerical solution of Euler's method to deal with the initial value problems for Erbium doped fiber amplifiers rate equations and gain dependences will be discussed. The paper interprets the theory of the erbium doped fiber amplifiers and analyzes photonic crystal fiber based erbium doped amplifiers. Simulation the propagation of the signal and pump power versus the amplifier length is to be done in this project.

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II. Erbium-Doped Fiber Amplifiers (EDFAs)

A. Three Levels Erbium-Doped Fiber Amplifiers (EDFAs)

The decay time from level 1 to the ground state normally is very short and can be neglected. So the rate equations for the three level N_3, N_2 and N_1, are presented as below:

$$\frac{dN_3}{dt} = -\frac{N_3}{\tau_{21}} + (N_1 - N_3)\phi_p\sigma_p \tag{1}$$

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau_{21}} + \frac{N_3}{\tau_{32}} - (N_2 - N_1)\phi_s\sigma_s \tag{2}$$

$$\frac{dN_3}{dt} = \frac{N_2}{\tau_{21}} - (N_1 - N_3)\phi_p\sigma_p + (N_2 - N_1)\phi_s\sigma_s \tag{3}$$

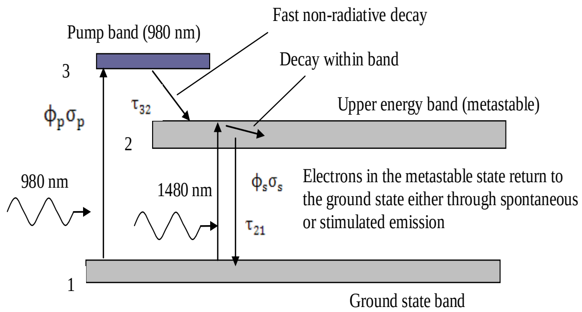


Fig. 1. The Three Level Energy Structure of the Erbium Er^{+3} Doped Fiber Amplifier.

Erbium is a rare earth element that is used for amplification operating around 1550 nm which is the wavelength for long haul communication system. Erbium doped optical silica-glass SiO_2 fiber makes EDFAs which are widely employed in optical communications. EDFAs which are using a unique, proprietary rare earth doping method and meet the requirements of high data transmission rates over long distances. They have played a crucial role in the advancement of communication system. The doped fiber is a gain medium in which signal and pump wavelengths are injected, and high-quantum-efficiency, high-rare earth concentration are produced. Amplification is obtained by interaction of wavelength and doping ions. The key physical phenomenon behind signal amplification is stimulated emission of radiation by atoms in the presence of an electromagnetic field. An optical signal in the case of optical amplifier is the field. For amplification to occur, population inversion has to be achieved by supplying additional energy in a suitable form to pump the electrons to the higher energy level. But to understand the impact of spontaneous emission is important.

B. Cross Sections

The emission cross section and the absorption cross section are related to the frequency or the wavelength. They express the abilities of an ion to either emit or absorb photon. If there are two energy levels E_1 and E_2 the energy difference is then the higher energy minus the lower energy E_2-E_1. And it is proportional to absorption cross section σ_{12} when the transition happens from level 1 to level 2. The emission cross section σ_{21} corresponds to the transition which takes place from level 2 to level 1 and is also proportional to the energy difference E_2-E_1. The relationship between the emission cross section and the absorption cross section is described by

$$\sigma_{21} = \sigma_{12}(\nu) e^{(g-h\nu)/kT} \tag{4}$$

where ε is the average energy of the manifolds.

C. Amplifier Gain

1) Stimulated emission is the process that causes the optical amplification. The optical gain G would take the form

$$P_{out}(t) = GP_{in}(t) \tag{5}$$

There are three classical ways of providing optical gain:

1) Optical power amplifier (or booster amplifier): amplifier immediately after transmitter to effectively increase transmitted power.
2) Optical preamplifier: amplifier immediately before receiver to increase received power and thus aid detection. It also effectively improves receiver sensitivity.
3) Optical in-line amplifier: amplifier periodically placed along the transmission line to compensate for fiber loss and often arranged in either pre- or booster configurations.

But it is more interesting to study the optical amplifier gain in the optical fibers. Assume that the fiber length is L, N is the density of photons. $\gamma(\nu)$ is the gain. Figure 2 states the amplifier gain at **z** position in the fiber.

$$d\phi = N * W * dz; W = \phi * \sigma(\nu) => d\phi = N * \phi * \sigma(\nu) * dz \tag{6}$$

$$\frac{d\phi(Z)}{dz} = \gamma(\nu)\phi(Z) = N\sigma(\nu)\phi(z) \tag{7}$$

The gain of the signal and the pump are yielded from:

$$g_s = (N_2 - N_1)\sigma_s \tag{8}$$

$$g_p = (N_3 - N_1)\sigma_p \tag{9}$$

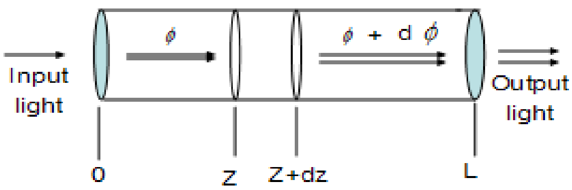


Fig.2. Amplifier Gain Diagram.

Considering a situation where the erbium ions are uniformly doped in the core of the PCF, the propagation equations which demonstrate the behavior of the erbium-doped fiber amplifiers can be rewritten as:

$$\frac{dI_s'(Z)}{dz} = \frac{1}{1 + I_s'(Z)/I_{sat}(Z)} \left(\frac{I'_p(Z) - 1}{I_p'(Z) + 1} \right)_s I_s'(Z)\sigma_s N \tag{10}$$

$$\frac{dI_p'(Z)}{dz} = -\frac{1 + \eta I_s'(Z)}{1 + 2\eta I_s'(Z) + I_p'(Z)} I_p'(Z)\sigma_p N \tag{11}$$

where $I_p' = \dfrac{I_p}{I_{th}}$; $I_s' = \dfrac{I_s}{I_{th}}$; are the normalized intensities in the units of the pump threshold. Where σ_s denotes absorption cross section when the transition happens from level 2 to level 1, and σ_p corresponds to the transition which takes place from level 1 to level 3.

Consider the field propagating along the fiber, take account infinitesimal length ***dz*** , and combine effects of pumping and stimulated emission:

$$\frac{d\phi_s}{dz} = \phi_s g_s \qquad \frac{d\phi_p}{dz} = \phi_p g_p \tag{12}$$

where $\phi_p = \dfrac{I_p}{h\nu_p}$, $\phi_s = \dfrac{I_s}{h\nu_s} g_s$, ν_p and ν_s are the frequencies of the pump and signal respectively. Eq. (8) and (9) will be modified as:

$$\frac{dI_s}{dz} = (N_2 - N_1)\sigma_s I_s \qquad \frac{dI_p}{dz} = (N_3 - N_1)\sigma_p I_p \tag{13}$$

D. Crystal Fiber Based EDFAs

For the photonic crystal based erbium doped holey core fiber amplifier, the contribution of the erbium ions is the complex refractive index in the core area. When a field propagates along the z axis in an amplifier, it can be described as:

$$E = E_0 \exp j(\omega t - \beta z) = E_0 \exp(j\omega t)\exp(-\beta z) \tag{14}$$

The wave equation is expressed as:

$$\nabla^2 E + n^2 k_0^2 E - \beta^2 E = 0 \tag{15}$$

Considering a situation where the erbium ions are uniformly doped in the core of the PCF, (10) and (11) will be changed as:

$$\frac{dI_s'(Z)}{dz} = \frac{1}{1 + I_s'(Z)/I_{sat}(Z)} \left(\frac{I'_p(Z) - 1}{I_p'(Z) + 1} \right) \Gamma_s I_s'(Z)\sigma_s N \tag{16}$$

$$\frac{dI_p'(Z)}{dz} = -\frac{1 + \eta I_s'(Z)}{1 + 2\eta I_s'(Z) + I_p'(Z)} \Gamma_p I_p'(Z)\sigma_p N \tag{17}$$

where $I_p' = \dfrac{I_p}{I_{th}}$; $I_s' = \dfrac{I_s}{I_{th}}$; $I_{th} = \dfrac{h\nu_p}{\sigma_p \tau_{21}}$;

$\eta = \dfrac{h\nu_p}{h\nu_s} \dfrac{\sigma_s}{\sigma_p}$; $I_{sat}(Z) = \dfrac{1 + I_p'(Z)}{2\eta}$; Γ_s and Γ_p are the confinement factors of the signal and pump respectively.

E. Euler Method for Photonic Crystal EDFAs

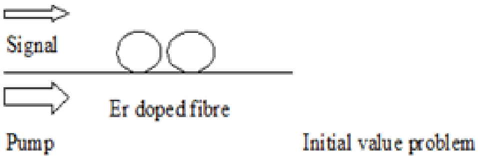


Fig. 3. Co-propagating pump

To solve the equations (14) and (15), one can use the Euler's method. Assume that equations have their unique solution and two continuous derivatives along the fiber length L, so that for each *k=0, 1, 2, 3,…, M-1,*

$$I_s^{'}(z_{k+1}) = I_s^{'}(z_k) + \Delta z \frac{dI_s^{'}(z_k)}{dz} + \Delta^2 z \frac{d^2 I_s^{'}(\xi_k)}{dz^2} \quad (18)$$

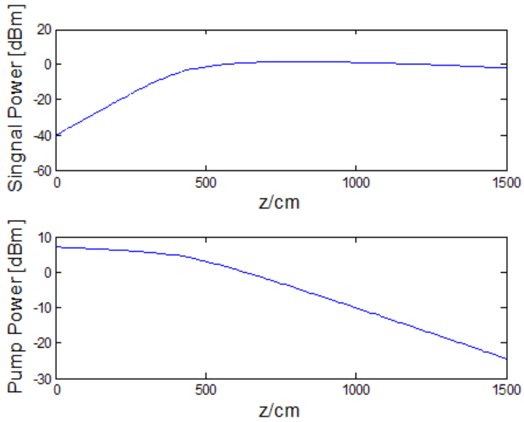
$$I_p^{'}(z_{k+1}) = I_p^{'}(z_k) + \Delta z \frac{dI_p^{'}(z_k)}{dz} + \Delta^2 z \frac{d^2 I_p^{'}(\xi_k)}{dz^2} \quad (19)$$

$z = z_0 + k\Delta z$, for each *k=0, 1, 2, 3…M.*

Hence, within the Normalized equations for PCF based erbium doped fiber amplifiers, choose proper parameters and initial values of signal and pump power, then exploit Euler's method, the output accurate value can be found.

III. Modeling Results

To demonstrate its effectiveness in the modal and the gain of the PC based EDFAs, the intersection number is 1000 and the mesh size is 1μm with the calculation window of 72×72 meshes. Both the confinement factors of the pump and signal are greater than 0.8. From the simulations, it can be found that if the initial pump power is less than 10dBm (see Fig4), the gain along the amplifier length will not be stable and there is a saturation gain existed. For the Er^{3+} concentration $N=0.7\times10^{19}/cm^{-3}$ we find the optimized gain is *G=54.813dB*, for initial pump power *Pp=50mW(17dBm)* and the initial signal power is Ps=-40dBm; see figure 5, and the optimal fiber length *L=14.7m*. When L=15m, the gain G=54.808dB. Pump power can boost the signal quickly and keep this amplification almost unchanged. In Ref. [5] for $N=0.2\times10^{19}/cm^{-3}$, it reported that the optimal fiber length is 18.6m; the optimized gain is G=41.9 dB under the same initial signal and pump power conditions.



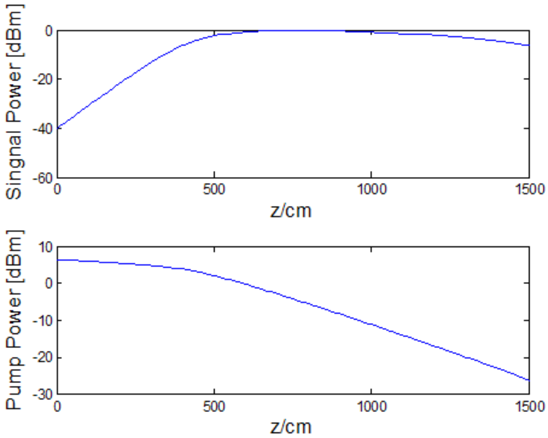


Fig. 4. (a)Gain= 22.287dB; Ps= -4.8dBm; Pp =10dBm. (b) gain= 33.94 dB; Ps=-40dBm; Pp=6.44dBm.

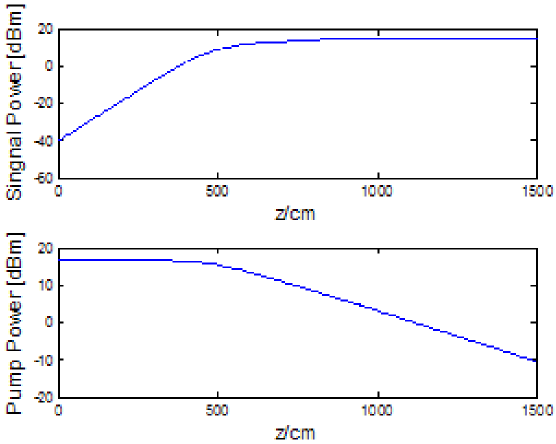


Fig. 5. PC based EDFA. The signal and the pump power profiles versus the amplifier length. G=54.81dB; Ps(0)=-40dBm; Pp(0)=16.99dBm(50mW).

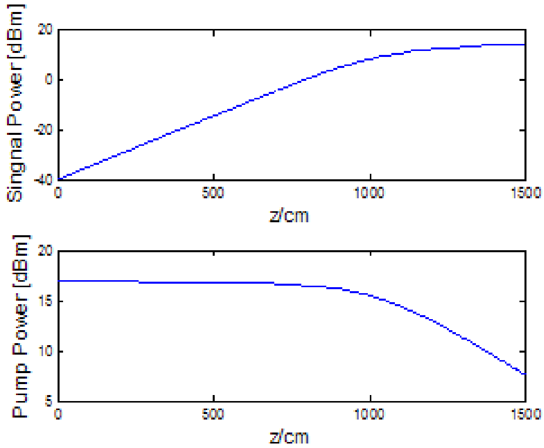


Fig. 6. Conventional EDFA. The signal and the pump power profiles versus the amplifier length. G=53.88dB; Ps(0)=-40dBm; Pp(0)=16.99dBm(50mW).

Fig.6 shows the input signal power Ps(0) = -40dBm and pump power Pp(0)=16.99dBm (50mW), the gain for this conventional EDFA is *G=53.88dB*. On the contrary, the PCF based erbium amplifiers can be very fast to boost the signal power, whereas the conventional EDFAs rather slow which is near to the end of the amplifier length.

IV. Conclusion

In this paper, it is found that the hollow-core PCF based erbium doped amplifiers of the length 15m, the relative core size *d/Λ=0.33,* when the input signal is -40dBm, the input pump power 50mW is 17dBm, the gain of this amplifier is obtained 54.81dB. The calculation window is used as $72\mu m \times 72\mu m$, the PCF core size $6\mu m$. It will be very interesting by using full vector method to calculate the solid core PCF based EDFAs gain profile. It is important to point out that the very promising properties of the PCF gives some new ideas, i.e. the PCF based Erbium Doped amplifiers would be the novel devices in the telecommunication systems. By employing Euler's methods, the gain profiles of the PCF based erbium doped amplifiers have been reported.

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