Study on the Sequence of Weakening Buffer Operator Based on the New Information

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***Abstract*—Based on the present theories of buffer operators, this paper proposed a kind of buffer operator, which all has the universality and practicability. We address their characteristics and their internal relations between these buffer operators. All these effectively resolves the questions , which often come forth in modeling forecasting of disturbed data sequence by shock wave and there are some contradictions between the quantity result and the qualitative analysis.**

***Index Terms*—Strictly monotonic function, weakening buffer operator, grey systems theory.**

I. Introduction

Grey system is good at dealing with uncertainty problems with characteristics of "small sample" and "poor information". And therefore using inherent information to mining the law of the system is basic principal of grey system. We can seek changing law between each factor or itself according to research social, economic, ecological, and other behavioral system data. In Grey theory's opinion, although with complex representation and disorderly data, system's overall function will inevitably contain some laws and the key is how to choose the appropriate ways to mining and use it [1-5]. In the literature [6-10], Professor Liu proposed the concept of shock disturbed operator and constructed a kind buffer operators that was applied widely. Based on those researches, combining with monotonic function, this paper constructed a new type of weakening buffer operator. The new operator can improve the accuracy in forecast modeling effectively.

II. Basic Concept

Definition 2.1: Assume that the sequence of data representing a system's behavior is given, $X=(x(1),x(2),x(3),\cdots x(n))$, then

1) X is called a monotonously increasing sequence if $\forall k=2,3,4,\cdots,n$, $x(k)-x(k-1)>0$.
2) X is called a monotonously decreasing sequence, if $\forall k=2,3,4,\cdots,n$, $x(k)-x(k-1)<0$.
3) X is called a vibration sequence if $k_1,k_2\in\{2,3,4,\cdots,n\}$, $x(k_1)-x(k_1-1)>0$ and $x(k_2)-x(k_2-1)<0$,

And $M=\max_{1\le k\le n} x(k)$, $m=\min_{1\le k\le n} x(k)$, then $M-m$ is called the amplitude of X.

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Definition 2.2: Assume that X is a sequence of raw data, D is an operator worked on X, and the sequence, obtained by having D worked on X, is denoted as

$$XD=(x(1)d,x(2)d,\cdots,x(n)d) \tag{1}$$

then, D is called a sequence operator, and XD the first order sequence worked on by the operator D. Sequence is referred to as operator.

A sequence operator can be applied as many times as needed. It can obtain a second order sequence, even order sequence, they can be denoted as $XD^2,\cdots,XD^r$.

Axiom 2.1: Axiom of Fixed Points [4]. Assume that X is a sequence of raw data and D is a sequence operator, then D must satisfy $x(k)d=x(n)$.

Axiom 2.2: Axiom on Sufficient Usage of Information [4]. When a sequence operator is applied, all the information contained in each datum $x(k),\ (k=1,2,3,\cdots,n)$ of the sequence X of the raw data should be sufficiently applied, and any effect of each entry $x(k),(k=1,2,3,\cdots,n)$ should also be directly reflected in the sequence worked on by the operator.

Axiom 2.3: Axiom of Analytic Representations [4]. For any $x(k)d,(k=1,2,3,\cdots,n)$ can be described with a uniform and elementary analytic representation in $x(1),x(2),x(3),\cdots x(n)$.

All sequence operators, satisfying these three axioms, are called buffer operators, XD is called buffer sequence.

Definition 2.3: Assume X is a sequence of raw data, D is an operator worked on X, when X is a monotonously increasing sequence, a monotonously decreasing sequence or a vibration sequence, if the buffer sequence XD increases or decrease more slowly or vibrate with a smaller amplitude than the original sequence X, the buffer operator D is termed as a weakening operator [5].

Theorem 2.1 [11-13]

1) When X is a monotonously increasing sequence, XD is a buffer sequence, then D is a weakening operator $\Leftrightarrow x(k)\le x(k)d$. $(k=1,2,3,\ldots,n)$.
2) When X is a monotonously decreasing sequence, XD is a buffer sequence, then, D is a weakening operator $\Leftrightarrow x(k)\ge x(k)d$, $(k=1,2,3,\ldots,n)$.
3) When X is a monotonously vibration sequence and D is

a weakening operator, XD is a buffer sequence, then,

$$\max_{1\le k\le n} x(k) \ge \max_{1\le k\le n}\{x(k)d\}$$

$$\max_{1\le k\le n} x(k) \le \max_{1\le k\le n}\{x(k)d\}$$

III. Weakening Buffer Operator Based on Strictly Monotone Function

Liu and Dang construct the following weakening buffer operators in its monograph [5]. Assume $X=(x(1),x(2),x(3),\cdots x(n))$ is a sequence of raw data, then

$$XD_1=(x(1)d_1,x(2)d_1,x(3)d_1,\cdots,x(n)d_1)$$

and

$$x(k)d_1=\frac{x(k)+x(k+1)+\cdots+x(n)}{n-k+1}$$

When X is a monotonously increasing sequence, a monotonously decreasing sequence or a vibration sequence, D_1 is a weakening operator. Here, we call D_1 as the average weakening buffer operator. And based on the operator, we construct a new weakening buffer operator through a strictly monotonic function.

Theorem 3.1: Assume $X=(x(1),x(2),x(3),\cdots x(n))$ is a sequence of raw data, $x_i>0, f_i>0$, f_i is a strictly monotonic function, g_i is its inverse function, and

$$XD_2=(x(1)d_2,x(2)d_2,x(3)d_2,\cdots,x(n)d_2).$$

$$x(k)d_2=g_i\{\frac{f_i(x(k))+\cdots+f(x(n))}{n-k+1}\}$$

When X is a monotonously increasing sequence, a monotonously decreasing sequence or a vibration sequence, D_2 is a weakening buffer operator.

Proof: First we consider that f_i is a monotonously increasing function, then, g_i is also a monotonously increasing function. Easily proved

$$x(n)d_2=g_i\left\{\frac{f_i(x(n))}{n-n+1}\right\}=x(n)$$

so XD_2 satisfies the three axioms, and D_2 is a buffer operator.

1) When X is a monotonously increasing sequence, because $0<x(k)\le\cdots\le x(n)$, so

$$0<f_i(x(k))\le\cdots\le f_i(x(n)),$$

$$0<(n-k+1)f_i(x(k))\le f_i(x(k))\cdots+f_i(x(n))$$

$$0<f_i(x(k))\le\frac{f_i(x(k))+\cdots+f_i(x(n))}{n-k+1}$$

$$x(k)d_2=g_i\left\{\frac{f_i(x(k)+\cdots f_i(x(n))}{n-k+1}\right\}\ge g_i(f_i(x(k)))=x(k)$$

so D_2 is a weakening buffer operator.

2) When X is a monotonously decreasing sequence, because $x(k)\ge\cdots\ge x(n)>0$, then

$$f_i(x(k))\ge\cdots\ge f_i(x(n))>0$$

$$(n-k+1)f_i(x(k))\ge f_i(x(k))+\cdots+f_i(x(n))>0$$

$$0<\frac{f_i(x(k))+\cdots+f_i(x(n))}{n-k+1}\le f_i(x(k))$$

$$x(k)d_2=g_i\left\{\frac{f_i(x(k)+\cdots f_i(x(n))}{n-k+1}\right\}\le g_i(f_i(x(k)))=x(k)$$

so D_2 is a weakening buffer operator.

3) When X is a vibration sequence, we can obtain the same result.

References

[1] J. L. Deng, "The grey exponential law of AGO," *Journal of Huazhong University of Science and Technology*, vol. 15, no. 5, pp.7-12, 1987.

[2] S. F. Liu, "The trap in the prediction of a shock disturbed system and the buffer operator," *Journal of Huazhong University of Science and Technology*, vol. 25, no. 1, pp.25-27, 1997.

[3] S. F. Liu, "Buffer operator and its application," *Theories and Practices of Grey System*, vol. 2, no. 1, pp. 45-50, 1992.

[4] S. F. Liu, "The three axioms of buffer operator and their application," *The Journal of Grey System*, vol. 3, no. 1, pp.39-48, 1991.

[5] S. F. Liu, Y. G. Dang, and Z. G. Fang, *Grey system theory and its application*, 3rd ed., Science Press, Beijing, 2004.

[6] Y. G. Dang, S. F. Liu, and B. Liu et al, "Study on the weakening buffer operators and researches," *Chinese Journal of Management Science*, vol. 12, no. 2, pp.108-111, 2004.

[7] Y. Q. Guan and S. F. Liu, "Study on the weakening buffer operator sequence," *Chinese Journal of Management Science*, vol. 15, no. 4, pp.89-92, 2007.

[8] Y. G. Dang, B. Liu, and Y. Q, Guan, "Study on the strengthening buffer operators," *Control and Decision*, vol. 20, no. 12, pp.1332-1336, 2007.

[9] Y. G. Dang, S. F. Liu, and C. M. Mi, "Study on characteristics of the strengthening buffer operators," *Control and Decision*, vol. 22, no. 7, pp.730-734, 2007.

[10] Y. Q. Guan and S. F. Liu, "Sequence of strengthening buffer operator and it's application based on fixed point," *Control and Decision*, vol. 22, no. 10, pp.1189-1192, 2007.

[11] Y. Q. Guan and S. F. Liu, "Study on the sequence of strengthening buffer operator and its application by m-thorder," *Journal of Yunnan Normal University*, vol. 27, no. 1, pp.32-35, 2007.

[12] D. A. Cao and S. M. Zhang, *Introduction to the cumulative-sum method*. Science Press, Beijing, 1999.

[13] N. M. Xie and S. F. Liu, "The character of strengthening buffer operator and building of some operators," *Statistics and Control*, vol. 37, no. 4, pp.9-10, 2006.